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A New Approach to Group Structure, Burden Sharing And the Equilibrium Provision of Public Goods*

by

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Abstract

In the received model of the voluntary provision of a pure public good, the usual practice is to proceed from assumptions about the group characteristics to inferences about an implied outcome. The approach advocated in this paper reverses the traditional direction. Assuming a Nash equilibrium, we ask how to characterize the diverse set of group characteristics which will support it. Approaching the problem from this angle we define three crucial characteristics of a group-equilibrium: consumer's "free rider inducing supply," "zero contribution-inducing wealth" and "voluntary surplus tribute" which is the amount by which a person's actual income exceeds his/her "zero-contribution inducing wealth." Defining these indicators we show how they form the foundation of a complete mapping between the distribution of individual characteristics of a group, and equilibrium public good supply. Certain questions such as the interaction between size of the group and heterogeneity of incomes and tastes not yet adequately addressed are shown to yield easily to this approach.

Keywords: Public goods; Voluntary provision; Collective action; Neutrality; Free riders; Altruism; Group heterogeneity, Burden sharing.

JEL Classification: H41.

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If any single issue represents the collective action dilemma faced by all groups from families to entire societies and collections of nations, it is the free rider problem. Self interested individual behavior following selfish individual incentives causes groups to fail to provide for themselves what it is in their own collective self-interest to do. At a practical level how uncoordinated decisions will cause burdens to be distributed is of highest concern in the provision of many international public goods such as global environmental protection, security, and financial stability (see Sandler 1992, 1997). Formalization of this problem from Olson (1965) onwards has yielded beautiful insights into the outcome to expect from various public good supplying groups and the nature of uncoordinated equilibrium (Shibata, 1971; Cornes and Sandler 1981, 1984; Bergstrom, Blume, and Varian, 1986). Included are characteristics of positive and zero contributors (Andreoni and McGuire, 1993), and effects of various changes in the composition and nature of the members of the group (Andreoni, 1988; Cornes and Hartley, 2002; Jack[1], 1991; McGuire, 1974), such as its total wealth, its distribution and so on. Yet the connection between the amount of public good provided, the total wealth of a group, the number of agents, their preferences, and the distribution of wealth among them and so on, seems to be recorded in a piecemeal and fragmented fashion in the literature.

This paper proposes a new concept around which we can organize equilibrium analysis of voluntary provision of pure public goods. Using this concept we organize and consolidate those results (some implicit and scattered) involving the effect of changes in membership count and diversity on the equilibrium provision of public good, on the pattern of positive contributors vs free riders, and on the distribution of voluntary contributions. This concept is derivative from that of "Free-Rider-Inducing Supply-Vector" (V-FRIS) proposed by McGuire (1991) and Andreoni and McGuire (1993). We call the new concept, the "Zero-Contribution-Inducing-Wealth Vector" (V-ZCIW). FRIS is defined as the aggregate supply of a public good

[1] This is Bryan C. Jack whose work in this area was cut short by his death on Sept 11, 2001, on the hi-jacked plane, which crashed into the Pentagon.

provided by others which is just sufficient to induce an individual to ride free and thus to contribute nothing. (Cornes and Hartley, 2002, in analysis contemporary with this call this amount the "drop-out" provision.) FRIS varies from person to person, depending on income and tastes. On the other hand, V-ZCIW is the vector of incomes --- one for each member of a group --- which for a common given aggregate public good supply (scalar) would cause each and every person in a group to free ride. Thus V-ZCIW is an imaginary, heuristic concept, since the ZCIW vector could not (even by chance) exist in reality unless the group received its public good as a donation from outside. Nevertheless, this concept will allow us to extend and simplify the analysis of the structure of group characteristics and Cournot-Nash public good supply.

The traditional approach to analysis of voluntary public good supply has been to proceed as follows:

1. Assume a given group structure for preferences, incomes, numbers etc.
2. Solve for the equilibrium provision of public good. (See Cornes and Hartley 2002, for a new, transparent, and general procedure for finding any equilibrium.)
3. Derive who contributes and how much. Identify the contributor set, etc.
4. Then ask how these equilibrium characteristics change if the assumptions change.

This procedure has been followed with much success. It has produced parts of the map from group population characteristics (numbers, income, tastes) to characteristics of the equilibrium supply of public good, including its quantity, identity of free riders, how much each contributor supplies etc. For example, this approach has given us the striking Shibata/Cornes-Sandler/Warr/Bergstrom-Blume-Varian result,[2] viz. that the aggregate supply of a public good, post-contribution individual private good consumptions, and post-contribution Nash-equilibrium utilities are invariant for any redistributions of income which do not alter the existing partition of the whole group into a “contributing set” of individuals and a “free riding set.” As we will see, using our notional or imaginary income vector, ZCIW, allows us to consolidate and extend these results. Inverting the problem, we will proceed in the opposite order.

[2] Although much of the current literature credits Warr (1983) for the discovery of the “neutrality”, this result also clearly follows from Shibata’s (1971) two-person bargaining triangle (p. 21-22). In addition well before Warr (1983) the Cornes and Sandler (1981) paper not only recognized the result but also was the first to indicate the bounds to neutrality (as referenced by Bergstrom, Blume and Varian, 1986, Footnote 3).

1. Assume a desired supply of public goods.

2. Ask what sets of population characteristics, incomes, tastes, etc are consistent with this supply being voluntarily provided as a Cournot-Nash Equilibrium.

3. Change the assumed supply, and find the new universe of sets of characteristics consistent with it being Cournot-Nash.

In extending the literature our approach will allow us to map the entire connection among the elements, population characteristics, amount of public good, individual contributions, zero contributing free riders etc. in a more organized, concise, and intuitive way than hitherto available. In doing this we also derive some interesting new results. We should emphasize that all results in this paper depend on equilibria being unique, and thus on an assumption that all goods are normal.

Groundwork: V-FRIS and Nash Equilibria

Free Rider Inducing Supply: For illustration consider a population of identical persons, identical tastes, and identical incomes --- with homothetic preferences (which guarantees normality), and therefore a linear Engel curves and Income Expansion Paths (IEPs). These latter pass through the origin and have slope γ_k.[3] Each individual has endowed income w_k. To repeat, here we first assume that γ_k has a common value for all k, and w_k has a common value for all k. Figure 1 shows one solitary person's provision --- his "isolation purchase" --- of the public good as $g_k^{n=1}$ (which in this case of a solitary individual is identical to provision by the entire group $G = G_{n=1}$). The figure also shows the homogeneous group's Nash equilibrium supply for one, two, up to n persons and the individual contributions of each person (see also Cornes and Sandler, 1996 for a similar diagrammatic analysis).[4] As n increases, each individual's contribution approaches zero and the

[3] This assumption simplifies exposition but is in no way essential. As we will see presently, without these restrictions, the rank order of individuals by their ZCIWs would be indeterminate; they could crisscross and meander so that different individuals might have the same ZCIW at some level of public good, G, but different ZCIW at other values of G.

[4] The horizontal distance from the y-axis to the individual's budget line gives one individual's contribution. When there are 2 persons, in equilibrium, the horizontal distance from the y-axis to the individual's budget line must equal the

aggregate contribution approaches G^0. This aggregate supply G^0 is also the amount which if given to the group freely whether to any solitary member of the group or to the group as a whole whatever its size --- out of the blue so to speak --- would cause *everyone* to free ride. Suppose we plot any individual's reaction curve, showing his individual contribution g_k versus everyone else's aggregate contribution, G_{-k}. Then when $G_{-k} = G^0$, $g_k = 0$. Thus G^0 is the scalar FRIS for a person with income w_k^0 and IEP of slope γ_k, (or the vector V-FRIS for a group of n identical individuals each with income w_k^0 and IEP of slope γ_k). As the diagram confirms, any individual's $FRIS_k$ is a function of his endowed income as well as his IEP_k or γ_k. If a group were composed of diverse individuals with varying IEP's and varying endowed incomes, each individual could be characterized by his own different FRIS along the lines of Figure 1.[5]

{Fig. 1. Goes Here}

Individual "Cut-off Income" and Voluntary Surplus Contribution: Now we want to call attention to a discovery due to Andreoni (1988) which will be important for our subsequent analysis. Andreoni showed that at a Nash Equilibrium, for any class of consumers of the same preferences (and therefore same values of γ_k) but variable incomes, there is a cut-off income/wealth (we use the terms interchangeably in this paper), such that those in any particular taste-class with income at or below their cut-off will free ride, while those in

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distance from the budget line to the IEP curve. Where this horizontal intersects the IEP gives the 2-person supply. (Indifference curves omitted throughout). When there are 3 persons the horizontal distance from the y-axis to the individual's budget line must equal ½ the distance from the budget line to the IEP curve, and the intersection determines the 3-person supply. Where there are n persons, the horizontal distance "d" from the y-axis to the individual's budget line must equal d = 1/(n – 1)th of the distance from the budget line to the IEP curve. As n increases without limit, d approaches zero, and total supply approaches G^0.

[5] We can also easily construct from Figure 1 the effect of adding new identical members to a group on the group's naïve Cournot reaction curve --- that is on the reaction curve of such a class taken as a whole, netting out all Cournot interactions between/among individual members as its membership, n, increases in number. As n increases without limit, this group's net reaction curve approaches a 45^O line (slope -1/1) with an x-intercept = y-intercept = FRIS of each (identical) individual member of the class. If no one else participates in a public good consuming population other than an infinite homogeneous class, then that infinite sized class will supply itself with the FRIS of any identical single member (each individual's supply approaching zero, but the aggregate of all individual contributions equal to FRIS as identified). And if some other non-identical outsider supplies this FRIS --- out of the blue so to speak --- then each individual in the homogeneous group will contribute zero so that the infinite group as a unit supplies zero. This establishes the two endpoints of the homogeneous group's reaction function, and since everything is linear, the function is simply the straight line connecting these two end points.

that class with income above the cut-off will spend their *entire income in excess of the cut-off* on provision of the public good. This cut-off depends on the characteristics of the group in question, their IEP's, their numbers, endowed incomes, i.e. all the characteristics that determine the Cournot-Nash equilibrium.

To illustrate this proposition we will at the same time show how the V-FRIS vector figures in the Cournot-Nash equilibrium of G and distribution of contributions when the distribution of tastes and incomes is non-uniform or heterogeneous. This example will serve as a preliminary exercise to our main results. Assume then that there are three individuals in a group with varying w_k and γ_k. Each individual has a different endowed income w_1, w_2, and w_3, and different marginal propensity to spend on the public good ("G") implicit in the different IEP[6] slopes γ_1, γ_2, and γ_3.

Figure 2 shows this initial position. Each person's FRIS is shown in the figure as G_1^0, G_2^0, and G_3^0. Start with the individual with the greatest FRIS, G_2^0. This happens to be Mr. 2. His "isolation purchase" --- the amount he will supply when no one else contributes or participates --- is given at point *B*. This amount exceeds G_1^0, Mr. 1's FRIS, but falls short of G_3^0. Therefore, if at the start only Mr. 2 contributes, then although Mr. 1 will not voluntarily contribute anything Mr. 3 will step in wanting to contribute. The Cournot-Nash equilibrium supply of 2 and 3 combined, therefore, must be derived. Without actually performing the derivation (a new and elegant derivation being contained in Cornes and Hartley, 2002) we will be satisfied with illustrating the solution, and just show this two person Nash Equilibrium at point *E*, where the amount G_{2+3} has been provided. Since $G_{2+3} > G_1^0$, Mr. 1 will continue to free ride exploiting the contributions of 2 and 3. At this outcome Mr. 2 provides amount g_2^*, at cost c_2^* in effect transferring this amount of his "Full Income" to Mr 3, while Mr. 3 provides amount g_3^*, at cost c_3^*, in effect transferring this amount of his "Full Income" to Mr. 2. The amount $(c_2^* + c_3^*)$ is also "transferred" to Mr. 1 so that his full income in Nash equilibrium becomes $w_1 + c_2^* + c_3^*$. With this full income, however, Mr 1 demands $G < G_{2+3}$ and, therefore, he does not contribute to the equilibrium supply so that $g_1^* = 0$.

[6] For slope of IEP = γ, the marginal propensity to spend on G is $1/(1 + \gamma)$.

Now notice in Figure 2 that for the actual equilibrium provision, Mr. 2's outlay c_2* reduces his presumed initial wealth of w_2 down to ω_2*; this is the hypothetical wealth for which Mr. 2 would free ride, given his preferences γ_2, and the total provision G_{2+3}. Similarly, c_3* reduces Mr. 3's wealth down to ω_3*, the hypothetical wealth which will cause Mr. 3 hypothetically to free ride given γ_3, and G_{2+3}. Thus ω_2* and ω_3* are the same as Andreoni's cut-off income, and as in Andreoni, each positive (non-free-riding) contributor spends the entire excess of his actual income over this cut-off on his voluntary provision. We will call this individual expenditure, viewed as an excess above a cut-off, a person's "Voluntary Surplus Tribute" (VST of Mr. k) to group needs, and will use s_k to indicate this amount. Thus, an individual's endowed wealth/income, zero-contribution income, and voluntary surplus tribute are related as:

"endowed wealth equals zero-contribution income plus voluntary tribute"

or:

$$w_k = \omega_k + s_k.$$

Note that if an individual rides free $s_k \leq 0$ and $w_k \leq \omega_k$. Thus, FRIS is the value of G which reduces VST just to zero (and g_k to zero) for any individual, or which makes $w_k = \omega_k$.[7]

{Fig. 2 Goes Here}

Identification and Use of V-ZCIW

With the fundamentals of FRIS and VST established we are now ready to turn to our inverse approach to the analysis of voluntary public good supply.[8]

Assumption: There are n persons in the group each with his own taste parameter γ_k, (k = 1…….n). There may be more than one person with the same taste, although in our illustrations we assume each person has his own unique taste parameter. Actual endowed incomes are not specified.

[7] As we have defined $VST_k = s_k$ it can be negative. We have also assumed that average cost of G is constant and equal to one the same for whoever contributes, so that if $s_k \geq 0$ then $s_k = g_k$. But since g_k cannot be negative, it follows that when $s_k < 0$, then $s_k \neq g_k$.

[8] We remind the reader that we assume normality of all goods, necessary for uniqueness of equilibrium.

Reminder: The cost of G^0 we take simply to be $C^0 = G^0$ itself since we assume constant average costs of unity across all contributors.[9]

Figure 3 gives picture of our approach with n = 3 and γ different for each person. We consider any arbitrary value of the public good, G^0. The vertical through G^0 intersects each IEP thereby determining each value of $\omega_k^o = f(\gamma_k, G^0)$, for k = 1…….n. In other words, this procedure identifies V-ZCIW as a function of G^0. Note again that V-ZCIW is a vector of wealths consistent with the heuristic assumption that G^0 is simultaneously the FRIS of each and every individual in the group. We say "heuristic" because $FRIS_k$ is just the minimum amount that *others* must supply to cause Mr. k to free ride. Therefore, V-ZCIW(G^0) identifies a vector of incomes for which G^0 is simultaneously the FRIS *for everyone*. But the combination [G^0, V-ZCIW(G^0)] could never be actually realized since with the wealth vector ω_k^o no one would actually contribute to G^0. The construct thus is purely heuristic and we cannot construe each person's "zero contribution" as depending on the *actual* contributions of others. To construe V-ZCIW in this way would be to write of $\omega_k^o = f(\gamma_k, G_{-k}^0)$, and this would be incorrect.

{Fig. 3 Goes Here}

Use of V-ZCIW and VST: Having identified ZCIW by the G^0 which induces it, our next step is to ask: "How can this $G^0 = C^0$ be financed in Cournot-Nash Equilibrium?" The answer to this question depends on properties of the vector of actual endowed incomes of the n people, i.e. on w_k and, in turn, on the vector of Voluntary Surplus Tributes, s_k^o defined above including its sum. Once we have identified all the distributions of income w_k^o (or of s_k^o) consistent with G^0, we will have complete mapping from that Nash Equilibrium provision to properties of the group. Without giving all possible details, we summarize this mapping with the following four propositions:

1a. Simple Failure of Nash Equilibrium: *If the total group income is $\Sigma_{k=1....n}\, \omega_k^0$ and it is distributed ω_1^0...... ω_k^0 .... ω_n^0, then G^0 cannot be financed internally from resources within the group.*

[9] See Ihori (1996) for the analysis in which contributors differ in their "productivities" or costs of contributing to

This is obvious since every one wants to ride free at this distribution of income and provision of G. We take this as a baseline for comparison with other distributions, and call it a baseline ZCIW vector.

1b. Failure of Nash Equilibrium by Replication: *If the population of the group increases m times by replication, from n to 2xn …….mxn, but the baseline distribution for each individual remains* ω_k^0 *(for each k = 1…n) as before, then public good provision* G^0 *continues to be unsupported as Nash equilibrium*.

2. Nash Equilibrium by Resource Injection: *If total group income is augmented to equal* $W = \Sigma_{k=1....n}\, \omega_k^0 + C^0$, *then to finance* $G^0 = C^0$, C^0 *can be distributed in any fashion whatsoever as* $C^0 = s_1^0 + ... s_k^0 +... ...+ s_n^0$ *provided* $s_k^0 \geq 0$ *for all k and, therefore, provided* $\omega_k^0 + s_k^0 \geq \omega_k^0$ *for all k.*

When the total injection of resources is C^0, any distribution of that total meeting the requirement that there be no transfers of pre-existing income among members of the group will guarantee that all group members are positive or marginally positive contributors.

3. Nash Equilibrium by Simple Redistribution: *If no additional resources are injected into the group, and none removed, then any redistribution of the existing baseline V-ZCIW resources* ω_k^0 $(k = 1.....n)$ *will sustain an equilibrium at* G^0 *provided* $\Sigma_{k=1....n}\, s_k^0 = 0$ *and provided* $\Sigma_{k=1....n} \left| s_k^0 \right| = 2C^0$ *and provided* $\omega_k^0 + s_k^0 = w_k^0 \geq 0$.

In other words, any redistribution of $\Sigma_{k=1....n}\, \omega_k^0$ which creates a sum of positive VST's equal to C^0 will sustain G^0.

4. Nash Equilibrium by Combined Redistribution and Injection: *Any combination of positive or negative injection plus redistributive transfers will support* G^0 *provided (a) for all positive* $s_k^0 > 0$, $\Sigma_k s_k^0 = C^0$ *and (b) for those* $s_k^0 \leq 0$; $0 \leq (s_k^0 + \omega_k^0) = w_k$.

public goods**.** This issue was also the main subject of Jack (1991).

This says that the sum of positive voluntary tributes of those individuals who do not free ride must just suffice to pay for G^0 and that the negative transfers going against free riders cannot be so great as to reduce their wealth below zero.

Although this approach to the structure of equilibrium in voluntary public good provision will not be utterly foreign to economists, it is different enough to cause a reader to ask: "why bother to understand it?" The merit of this approach is a combination of its generality we believe and its simplicity. Combined with the results of Andreoni, Bergstrom et al., Cornes-Sandler, Fries et al. and others it makes the structure of Nash-Cournot equilibrium transparent for just about any case imaginable, including the one in which the membership of any one of the taste-income classes (with a given FRIS) increases.[10] This is clear from Figure 3, which can be drawn for any population, of any size and diversity, and of any composition and total of incomes. Moreover, our approach allows us to determine the effects of changes in group characteristics on the equilibrium G and the distribution of its burden without solving for the new Cournot-Nash equilibrium all over again. The new equilibrium G and the distribution of its burden can be determined simply from the knowledge of the old G supplied before such changes in group characteristics. To further motivate this approach**,** we will next develop several applications.

Further Implications and Extensions

Our approach weaves together the concepts of FRIS, ZCIW and VST to demonstrate how they form the foundation for a complete mapping between group characteristic and the desired level of the public good to be supported as the Nash equilibrium. The advantages of this approach, we believe, are manifold. First, it directs attention toward the difficult problem of identifying the group configurations that support a given public good supply. As shown by Fig. 3, for each of the continuum of values of G^0 an infinite diversity of

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[10] Within the framework of Andreoni, or of Fries et al. and of others, it can be difficult to analyze the structure of equilibrium for many potential population configurations. Especially troublesome is deriving the consequences when the membership of only one sub-class within a diverse population changes. As we show our approach identifies such outcomes directly, leading us to entirely new results, hitherto buried in previous analyses and not recognized**.**

equilibrium sustaining endowed income distributions is possible.[11] Second, once we know the Cournot-Nash solution, G^0, for a given population of any taste distribution and any income distribution, we can apply the concepts of ZCIW, FRIS, and VST to analyze the effect of changes in group configurations on the Nash equilibrium G and distribution of its burden among the group members. The applications of these concepts also yield easy evaluation of the welfare effects of such changes in group characteristics. The analysis is more intuitive, simpler and yet more comprehensive than hitherto available in that it consolidates the entire spectrum of results such as those of Andreoni, 1988; Bergstrom et al., 1986; Cornes and Sandler 1984, 2000; Fries et al., 1991; McGuire and Groth, 1985; Shrestha, 2002). In particular, we analyze:

(1) *Effects of Adding One Agent of Different Taste Group*: First, let the population be increased by one person with a particular income and taste, without changing the total magnitude or the dispersion of tastes or wealth among the original population. Let the additional agent be labelled by "m". What effect does the introduction of the additional agent (with his income and tastes) have on the new equilibrium value of G and on the division of voluntary taxes/tributes among the original "n" agents? To answer this maintain the amount of G provisionally at the original G^0 and then calculate Mr. m's VST,[12] call this "s_m^0," for this original G^0. Now we state the following effects as obvious from Figure 3.

If $s_m^0 \leq 0$, Mr. m should be grouped with all the other non-contributors. His introduction has no effect on the outcome.

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[11] We leave it to mathematicians to determine the numerosity or magnitude of these configurations. Nevertheless, we present a few such typical configurations viz: the equilibria in which (1) all consumers are contributors; this occurs when all the consumers from the group have equal *positive* VSTs such that $\sum_k s_k = C^0$ for all k; or (2) a single consumer is a contributor, this occurs when one of the consumers VST is sufficient to finance G^0 (i.e., $s_k = C^0$ for one k) and all other $s_{j\neq k} \leq 0$; or (3) the richest consumer as a free rider, this occurs when the richest consumer k has sufficiently weak preference for G (reflected in higher γ_k) such that $\omega_k^0 \geq w_k$.

[12] The characteristics of Mr. m (i.e., w_m and γ_m) are completely arbitrary. He may be like one member of the existing group-n or different from all of them. Thus m and his derived VST at the original G, s_m^0, are exogenous. Notice that $s_m^0 = w_m - \omega_m^0$, where ω_m^0 is a function of G^0 and γ_m. It is also clear that s_m^0 can either be positive, negative or zero, depending upon the value of G^0 (and the characteristics of m).

If $s_m^0 > 0$, then adding Mr. m will have an effect on the outcome. First, it increases equilibrium G above G^0, and necessarily increases aggregate voluntary contributions since more G costs more. It influences (increases) the new equilibrium welfare of those original agents with $s_k^0 \leq 0$, but has no effect on their voluntary contributions, which remain nil. For those original k-agents with $0 < s_k^0 < s_m^0$, the introduction of Mr. m reduces their contributions, possibly down to zero, and increases their welfare. For those agents with $s_k^0 > s_m^0$, the introduction of Mr. m increases their welfare and reduces their contributions but never down to zero. For any agents with $s_k^0 = s_m^0$ the introduction of Mr. m will increase their welfare, and will reduce their contributions possibly to approach zero.[13]

(2) *Effect of an Arbitrary Increase in Total Membership Count as a Whole*: Now suppose that this "initial" population supporting G^0 changes in any manner whatsoever, different total count of individuals (now "m" rather than "n") and/or different distribution of characteristics and incomes now designated by "*".

Of course we can always rank order this new population by $FRIS_{k=1....m}$ --- however diverse its mix of utility functions and IEP's --- then use Andreoni-McGuire (1993) to find the new Cournot-Nash solution G*, and then go on further as shown in this paper to derive, given w_k, the details of ω_k and s_k at that value of G*. But rather than apply this brute force attack, our approach suggests more finesse: for the same old value of G^0 now calculate a new value of $C^* = \Sigma_{k=1....m}\, s_k^*$ for all $s_k^* > 0$, (where $s_k^* = w_k - \omega_k(\gamma_k, G^0)$ for all k = 1….m). If $C^* > G^0$ then G must be increased above G^0 to find the new Cournot-Nash equilibrium. Since all $\gamma_k > 0$ and ω_k is an increasing function of G, as G is increased above G^0 the new calculated values of C are less than C*. Thus, as G increases C declines. Therefore, continue to increase G until the two are brought into equality, i.e., $C^*(G^*) = G^*$. Once the new G* is found, s_k^* for all k = 1….m which are consistent with this new G*, and hence with the new set of contributors or free riders, is easily determined. Symmetrically if $C^* < G^0$ (this happens when the membership count of the group declines) the new equilibrium G declines.

[13] See Cornes and Sandler (2000) for analysis of how welfare of both contributors and free riders changes when income is redistributed from free riders to contributors depending on the number of free riders and the "contribution-

(3) *Effects of Replication:* To derive the effect of replication, begin with the initial population configuration and its Nash equilibrium. At this starting point rank order the members of the group by each's FRIS, (and take note of their VST's). Assume (for illustration only) that for each taste class there is one individual agent with wealth w_k, and suppose that everyone who contributes (whose VST = $s_k^0 > 0$) can be placed in one sub-group, and that everyone who does not (whose VST = $s_k^0 \leq 0$) can be placed in another sub-group. These sub-groupings of individuals or single-member classes are independent of the rank ordering by FRIS.[14] From this initial position now let us "replicate." We consider two cases:

First Case: Replication of Only One Class. First suppose just one single member class, j, with its associated γ_j and w_j were doubled, tripled, etc. in size, but all other classes remained of the same membership containing only one person. Note that neither Andreoni nor Fries et al. consider this case; within their frameworks its analysis is awkward. With the help of Figure 4, however, we can easily observe:

If the original $s_j \leq 0$, then replication of j has no effect at all.

If $s_j > 0$, then the replication of j will increase G^0. As the number of replications of j increases, all contributors "*below*" j are driven to a zero VST, (where "below" means having lower FRIS).

If there are no classes "above" class j --- i.e. with a higher FRIS --- then as the count of the members of class j increases without limit, each of that class's member's VST approaches zero, while the total (provided by all contributors from that class j together) Nash equilibrium supply of G approaches $G^* = FRIS_j = G_j^0$.

On the other hand if classes "above" j do exist say class "i" (with $G_i^0 > G_j^0$) then as class j replicates, total group supply (provided by all contributors in all classes together) approaches $G^* = G_j^0$, with implied cost C*. But all members of classes i now contribute Σ [$VST_i(G^*)$], which leaves the members of class j (who are infinite in number) with provision or cost [C* - Σ (VST_i)] to divide

responsiveness" of the positive contributors.

[14] For purposes of visualization we might revert to an assumption that individuals can be rank ordered by their tastes, that is by γ_k; this ordering will be identical to an ordering by ZCIW, $\omega^o{}_k$, and will be the same for all values of G^0. This is not at all necessary but it may be helpful to clarify the argument. It might also be helpful though not necessary to show some regular relationship between γ_k and w_k, such that w_k increases or decreases uniformly with γ_k. Doing this would allow a simple correspondence between FRIS, taste parameter, and endowed wealth.

up equally among themselves. This result is quite distinct from that obtained by Andreoni (1988) and Fries et al. (1991). In their analyses the membership of each and every sub-class within a group increases by replication. In their case as the total group membership count grows to infinity (or very high, but finite, level), only the one sub-class with the highest FRIS bears the entire burden. But as our analysis demonstrates, a more discriminatory selective replication yields a different outcome altogether, even though the total group membership count increases without limit in both situations.

We can use Figure 4 to illustrate and even derive these conclusions. Figure 4a constructs a two-person (one Mr. 1 and one Mr. 2) reaction curve system, showing the stable equilibrium at their intersection. In this figure, f_1 denotes Mr. 1's isolation purchase and f_2 that of Mr. 2.

{Figures 4a. through 4c. here}

Now we want to use this diagram to understand how the equilibrium changes when one type or the other, type-1 or type-2, begins to replicate. The reaction curve of the type that does not increase in number stays the same, so we need to know only the effect of replication on the reaction curve of the type that increases. First let new entrants identical to Mr. 1 and n_1 in number come into the system. As the number n_1 increases without limit the reaction curve of the Mr-1-types taken as a group (i.e. net of all their Cournot interaction with each other) approaches a 45^O degree line (slope -1/1) which passes through an x-intercept = G_1^0 and a y-intercept = G_1^0. As explained above in connection with Figure 1, if outside parties provide nothing, the infinite sized sub-class of type-1 will supply itself with the $G_1^{\mathbf{0}}$ of any of its identical individual members. And if an outside party supplies G_1^0 then each individual in n_1 (of infinite count) will contribute absolutely nothing. The reaction curve, $R^{inf}{}_1$, of the infinitely replicated group of type-1 simply connects these two end points in a straight line. As shown in Figure 4b. when sub-class of type-1 replicates, the reaction curve through (G_1^0, G_1^0) lies entirely outside that of Mr. 2, R_2, so that the new solution is at the corner where only type-1 contributes, in the aggregate amount G_1^0.

On the other hand, if it is type 2 that replicates, number n_2 increases without limit and type 1 remains a single Mr. 1, then the reaction curve of this group, R^{inf}_2 , is shown in Figure 4c. Here an interior solution results at the intersection of the two reaction functions, the amount $FRIS_2 = G_2^0$ is provided but (the infinite sized) sub-class of 2-type provides only a fraction of G_2^0; a portion is provided by Mr. 1 (who has the higher governing $FRIS_1 = G_1^0 > FRIS_2 = G_2^0$).

Obviously this analysis of replicating one class can be pursued sequentially to infer the outcomes if first one class is increased in size, then in addition another is increased, then in addition another, etc..

Second Case: Replication of The Entire Population. Now we can turn to the case where we replicate all classes “simultaneously" beginning with n classes each of a single member. Actually this case is the simpler of the two. It is equivalent to the cases analyzed by Fries et al. (1991) and confirms his result. As the size of each class doubles, then triples etc, the Nash equilibrium provision of G for the entire growing population increases. From any intermediate equilibrium with its “own” G*, adding a replication simply increases Σ ($VST_{k = 1…n}$) --- where “k” indicates all classes and Σ the number of replications. Therefore, the new Σ ($VST_{k = 1…n}$) = C** at the “old” G* is more than sufficient to pay for G*, which must therefore increase. But for this replication, as G increases C (equal to the sum of VST's) declines; the new equilibrium G** for this particular number of replications is found where C**(G**) = G**. As replication continues, G** continues to increase and consequently “lower” classes (call them again say j-classes) find that $G^{**} \geq FRIS_j$ and eventually that their $VST_j \leq 0$. Therefore, these sub-classes drop out of contributing anything.

This cascade of free riding proceeds from one group to the next up the FRIS chain, until only the group with the highest FRIS remains to contribute. Thus, confirming Andreoni’s and Fries et al.’s conclusion, as each and every sub-group or class of the entire population is replicated equally, all classes progressively drop out of making any contribution at all, leaving only the class with the highest FRIS to carry the entire burden.

This is the class in an operational sense with the "highest demand." But as our analysis shows, this highest demanding class will only carry the entire burden if its numbers grow *pari passu* with increases in the rest of the society.

If, however, as shown above, membership count of some class "lower down" the (FRIS) demand scale increases without limit while memberships of classes higher up do not increase, then it is the FRIS of the lower class that determines equilibrium G of upper class and of the entire group. This result is clearly different from that obtained by both Andreoni and Fries et al. Unlike their results, ours shows that the one class with the highest FRIS may *not* necessarily be left to bear the entire burden of G even when the membership count of the group as a whole increases without limit. It all depends on how such membership count grows---by replication of the entire population or the replication of only one taste class of smaller-than-the-maximum FRIS.

Conclusions

This paper takes a new approach to voluntary public good provision that begins by assuming a Nash equilibrium level of a public good *a priori* and then characterizes the highly diverse universe of groups of consumers that can support this equilibrium. The concept of voluntary surplus tribute allows an easy identification of the distribution of burden (in financing the predetermined Nash level of the public good) among the group members. In addition, not only is one able easily to map the universe of group configurations into equilibrium, but from this new viewpoint the traditional problem --- moving from changes in group configuration to changes in Nash equilibrium supply and distribution of burden --- becomes more transparent. Our approach complements other recent work to consolidate and simplify our understanding of this central problem in public economics.

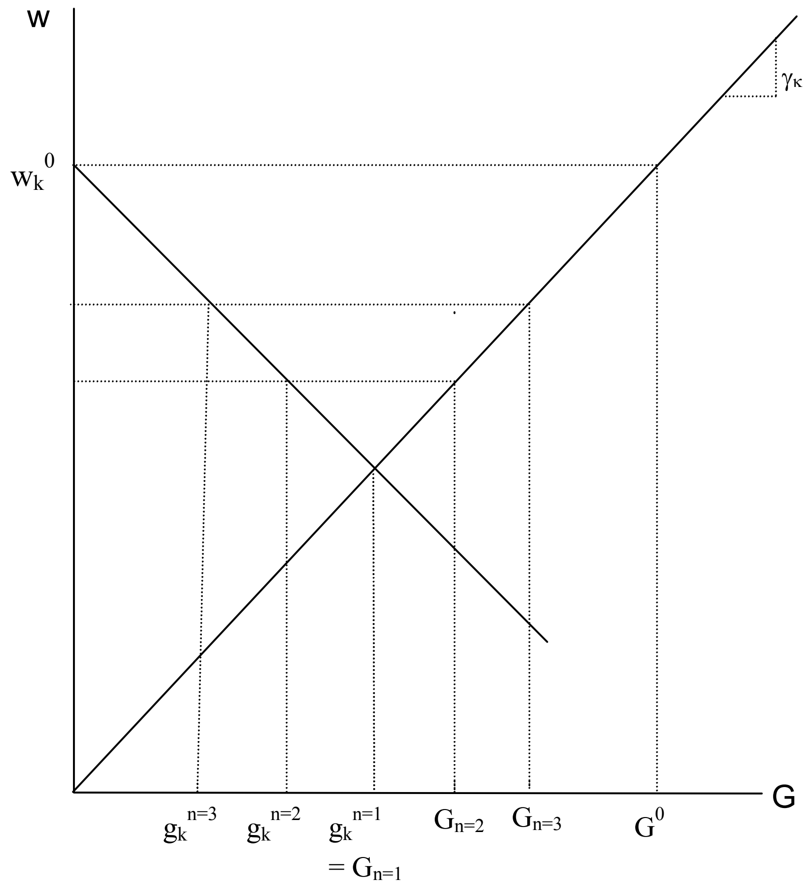

Fig. 1: FRIS and Individual Contributions for an Expanding Homogenous Group.

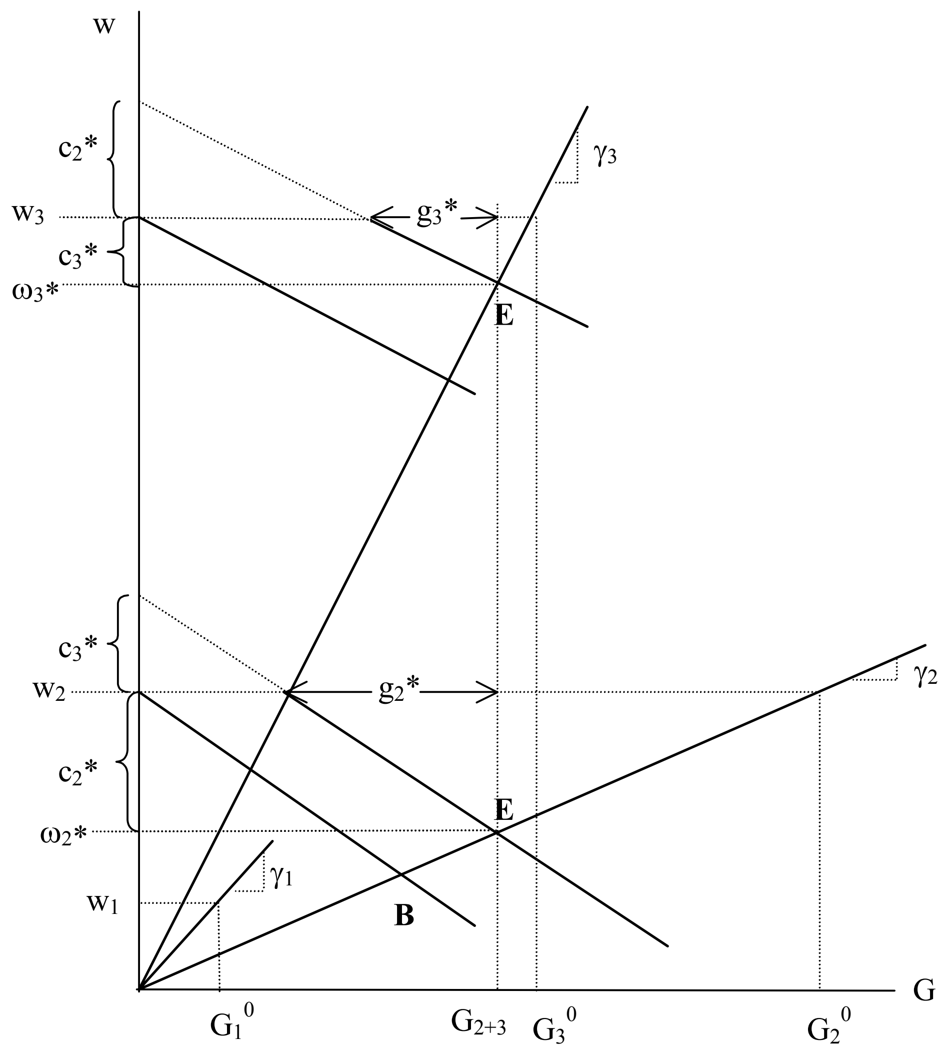

Fig. 2: FRIS, Isolation Purchase, and Equilibrium Contributions:
A Case with Two Contributors and One Free Rider.

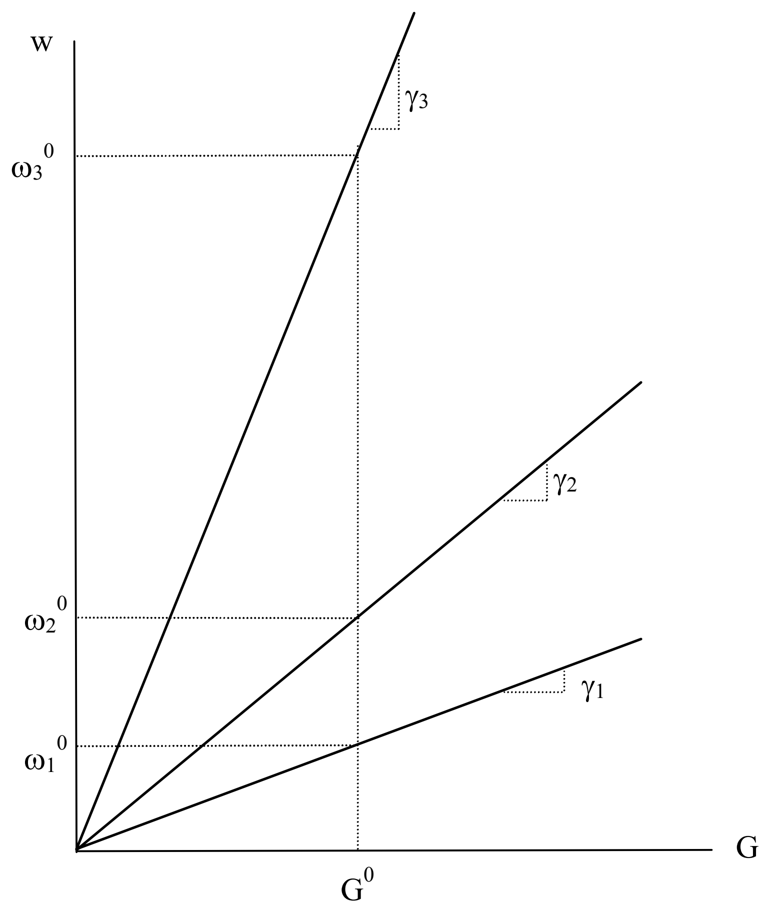

Fig. 3: V-ZCIW for an Arbitrary G^0.

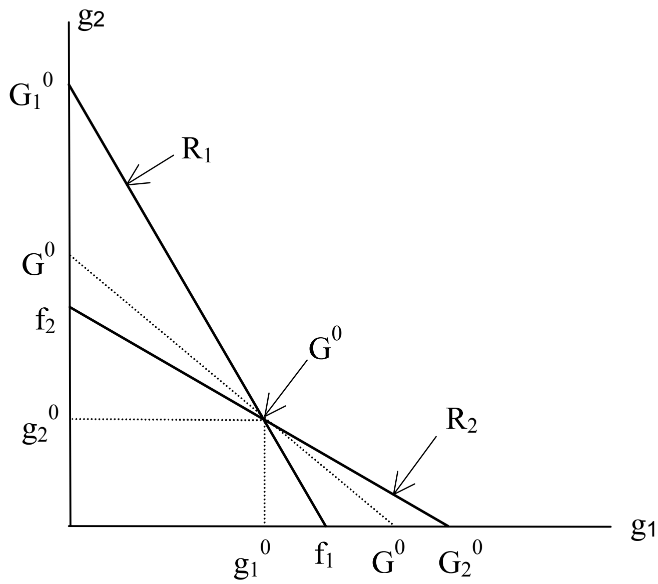

Fig. 4(a): Two-Agent Nash Equilibrium

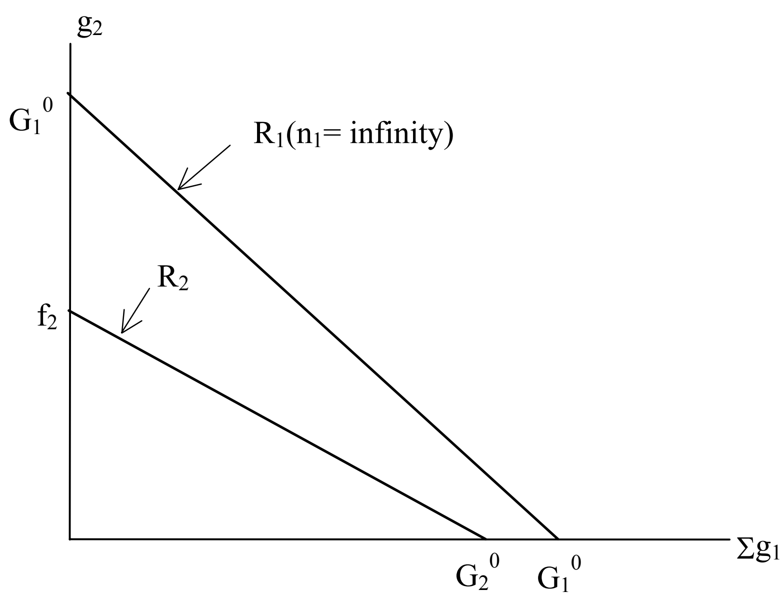

Fig. 4(b): Nash Equilibrium for $n_1 = \propto$ and $n_2 = 1$.

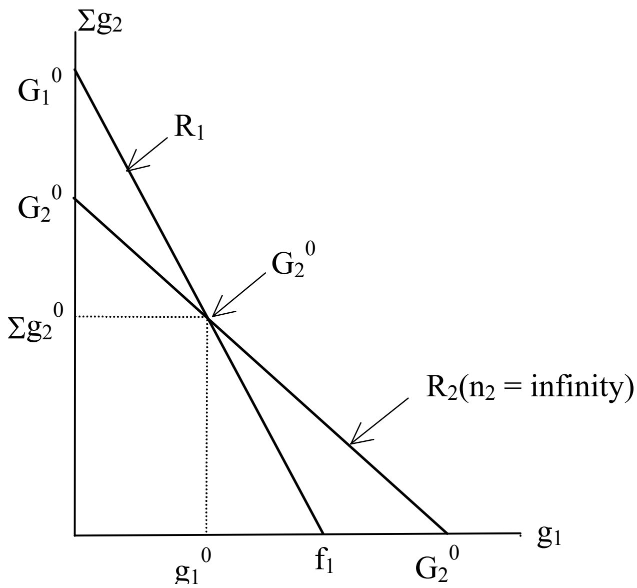

Fig. 4(c): Nash Equilibrium for $n_2 = \propto$ and $n_1 = 1$.

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