Discrete Public Goods under Threshold Uncertainty: Theory and Experiment

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Abstract: A discrete public good is provided when total contributions exceed the contribution threshold. I show that for a large class of threshold probability distributions, an increase in threshold uncertainty by 2nd-order stochastic dominance will increase (decrease) equilibrium contributions when the public good value is sufficiently high (low). In an experiment designed to test these predictions, behavior only moderately verifies the predictions. Using elicited beliefs data to represent subjects' beliefs, I find that behavior is not consistent with expected payoff maximization, however, contributions are increasing in subjects' subjective pivotalness. Thus, wider threshold uncertainty will sometimes—but not always—hinder collective action.

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1 Introduction

The ability or inability of groups to act collectively explains a variety of social phenomena, from cartels to military coups, from economic policies to social norms, and from political institutions to social activism. As such, researchers seeking to explain these and other phenomena have looked for preconditions for successful collective action. Since Olson's (1965) seminal work, economists have examined how a number of factors, such as group size, excludability, selective incentives, punishment, and so on, inhibit or foster collective.[2] One factor that potentially affects individuals' decisions to participate in a collective action is uncertainty about the *threshold* level of contributions needed for successful action. For example, neighborhood residents will not know how many individual requests to City Hall will be required to install a traffic signal at a local intersection; citizens might not know the amount of money required to complete a public project; or, more dramatically, coup plotters will not know how big their faction needs to be to overthrow the incumbent dictator.

Previous research has acknowledged that threshold (or provision) uncertainty can affect collective action. Nitzan and Romano (1990) conduct the first theoretical study of the topic. They model the setting as a discrete (step-level) public good game in which the threshold is chosen randomly from a commonly known threshold distribution and where voluntary[3] contributions are made simultaneously. They find that initial increases in threshold uncertainty (e.g., a wider variance in the threshold distribution) will often, but not always, be matched by increased contributions, and that efficient equilibria often still. A second theoretical study by Suleiman (1997) finds a similar result for the simplified game with a uni-

[2]There are many strands of collective action research. See Sandler (1992) for general presentation of collective action from the economics perspective. Another formal approach different from the discrete public good analysis used here that is often by sociologists to explain riots and revolutions is the *threshold* or *tipping* model, as in Granovetter (1978), Yin (1998), and Chwe (1999, 2000). A different methodological approach to study collective action is one that focuses on authority and structure (Lichbach 1998).

[3]The focus on voluntary participation can be justified. By leaving aside other possible factors (e.g., punishment), this focus allows for identifying the fundamental effect of threshold uncertainty on individuals' incentives to participate in an institution-free environment. This serves as a useful benchmark and can provide direction for future researchers to identify how groups can or should act to overcome or complement these effects. Moreover, there is already an established body of research that focuses on voluntary participation collective action as discrete public good games, so my findings can compare directly with existing findings.

form threshold distribution. There have also been experimental studies. Wit and Wilke's (1998) and Au's (2004) conducted experiments with sequential contributions, and they find that contribution levels are lower under higher threshold uncertainty. Gustafsson, Biel, and Gärling (1998) report a similar finding in an analogous experiment with simultaneous contributions. Finally, Suleiman, Budescu, and Rapoport (2001) find in a simultaneous contributions experiment that the effect of threshold uncertainty can depend on the mean of the threshold distribution. When the mean is low, increasing uncertainty can increase contributions, but the reverse holds when the mean is high.

This paper presents a theoretical and experimental study of an unexplored aspect of this issue—how the effect of increased threshold uncertainty on contributions depends on the value of the public good. My theoretical analysis yields a preliminary prediction: a small increase in threshold uncertainty will actually increase voluntary participation when individuals' benefits of successful action are sufficiently large, but it will decrease participation when the benefits are small. Like Nitzan and Romano (1990), I obtain my findings by building on Palfrey and Rosenthal (1984), who first modeled collective action as voluntary contributions towards a discrete public good. They show that with a commonly known threshold there always exists an efficient (Nash) equilibrium, although inefficient equilibria can also exist. I add threshold uncertainty to their model and examine how (Bayesian Nash) equilibrium contributions change as the threshold uncertainty changes. In particular, when the value of the public good is sufficiently large, an increase in threshold uncertainty (i.e., by second order stochastic dominance) increases an individual's probability of being a pivotal contributor, thereby increasing the probability she will contribute and driving up equilibrium contributions. When the value of the public good is small, the uncertainty increase will decrease the probability of being pivotal.

I test these predictions using data collected from a laboratory experiment. My experiment, although not the first to vary threshold uncertainty (see references above), is the first to vary both threshold uncertainty and the value of the public good. Thus, it provides me with the best available data to test the above predictions. Overall, the data moderately verify those predictions: contribution levels under one threshold probability distribution usually differ from contributions under another distribution as qualitatively predicted by the theoretical model. However, the predictions are not always verified. Two questions follow: why are the predictions verified to any degree, and why is that level of verification so weak? I address these questions using data on subjects' elicited beliefs about other players' contribution levels. An examination of these data reveals that subjects do update their reported beliefs in manners consistent with many learning models. This finding justifies using these data to proxy for subjects' true beliefs, which in turn allows me to calculate subjects' implied subjective probabilities of being pivotal. Conditioning on this subjective pivotalness, I show that subjects do not behave in a manner consistent with the model's implied decision rule. While this fact weakens the predictive power of the model, an important qualitative feature of the model is observed: contributions are increasing in subjects' subjective pivotalness. Thus, when making contribution decisions, individuals do think strategically in that they care about pivotalness.

The overall conclusion is that threshold uncertainty does not necessarily inhibit collective action for two reasons: first, individuals are successful, to some degree, in relating changes in threshold uncertainty into changes in subjective beliefs about pivotalness, and second, individuals do respond to pivotalness, even if not to the degree implied by the model. However, threshold uncertainty in some settings—particularly when the public good is low-valued—is likely to hinder collective action. This has both positive and negative implications concerning actual groups' abilities to act collectively. Successful collective action may occur in the face of threshold uncertainty even when participation is voluntary and uncoordinated, however, when the value of the public good is low, a potentially costly reduction in threshold uncertainty may be necessary for successful action unless there are other mitigating factors (e.g., selective incentives, punishment, etc.).

This paper fits best in the literature that considers the role of uncertainty in collective action. Previous work has examined, among other things, uncertainty about other individuals' altruism (Palfrey and Rosenthal 1988), uncertainty about other individuals' contribution costs (Palfrey and Rosenthal 1991), and uncertainty about other individuals' valuations of the public good (Menezes, Monteiro, and Temimi 2001).[4] My theoretical work is closest to Nitzan and Romano (1990), the difference being that I have each player make a binary contribution choice, which better represents collective action scenarios that involve participation, in-out, or yes-no decisions instead of continuous contributions, which better represent monetary contributions. I show how this difference has efficiency implications. Finally, my work is also closely related to the theoretical and experimental work on common pool resources (CPRs) with unknown pool size (e.g., see Budescu, Rapoport, and Suleiman 1995).

2 Model

The discrete public good game consists of the following. The set of expected payoff maximizing players is $N = \{1, ..., n\}$, $2 < n < \infty$. Players have identical binary action sets $A_i = \{0, 1\}$, with actions labeled $\{\text{don't contribute, contribute}\}$. When players mix over those actions, let α_i be the probability that $a_i = 1$ (i contributes). The cost of contributing c and the value of a provided public good v are the same for all individuals. The contribution threshold t to provide the public good is chosen from a publicly known distribution cdf F with pdf f s.t. $F(0) = 0$. Given C realized contributions, the payoffs are:

$$\text{payoff for } i = \begin{cases} v & \text{if } \; C \geq t \text{ and } a_i = 0 \\ v - c & \text{if } \; C \geq t \text{ and } a_i = 1 \\ 0 & \text{if } \; C < t \text{ and } a_i = 0 \\ -c & \text{if } \; C < t \text{ and } a_i = 1 \end{cases}.$$

For most of this paper I use discrete threshold distributions, but assuming an underlying continuous contribution will be necessary to consider what happens when the binary action set assumption is relaxed in Section 3.4.[5]

The timing of the game is as follows: (1) n, v, c, F, and the game set-up are commonly known; (2) the players simultaneously choose whether or not to contribute; (3) payoffs are received. The analysis focuses on Bayesian Nash equilibria.

[4]Palfrey and Rosenthal (1988, 1991) examine discrete contributions. Menezes, Monteiro, and Temimi (2001) build on Bagnoli and Lipman (1989) in considering continuous contributions.

[5]The analysis is unchanged if the discrete F is actually a discrete representation of an underlying continuous threshold distribution. For example, consider a continuous cdf $H(x)$ with pdf $h(x)$ over $(0, \infty)$. Then F the discrete version of H (with pdf $h(x)$) is calculated s.t. $F(x) = \int_0^x h(x)dx$ for all $x = \{1, 2, ...\}$.

3 Analysis

The number of contributions in equilibrium is of greater interest than which players contribute in equilibrium. I will treat two equilibria with the same number of contributions as a unique equilibrium.

Sections 3.1-3 assume that the threshold distribution is strictly quasi-concave. Formally, denote cdf F *strictly quasi-concave* if its pdf is single-peaked and if $f(x) \neq f(x+1)$ for any $f(x) > 0$, while the pdf can be flat at any x where $f(x) = 0$.[6] The main results in this paper can be obtained without this condition (see Section 3.4), but this condition is not unrealistic and greatly simplifies the analysis.

3.1 Pure Equilibria

For now, the focus is on pure equilibria. An agent's decision in equilibrium will depend on the probability she is pivotal in providing the public good. Denote C_{-i} to be the set of contributing agents besides i. The payoff matrix is

	$C_{-i} < t-1$ (lost cause)	$C_{-i} = t-1$ (pivotal)	$C_{-i} > t-1$ (redundant)
spend	$-c$	$v-c$	$v-c$
keep	0	0	v

Let $a_{-i} = (a_1, ..., a_{i-1}, a_{i+1}, ..., a_n)$. Further denote $\Pr[piv|a_{-i}, F]$ the probability that i is pivotal given a_{-i} and F, $\Pr[lost|a_{-i}, F]$ the probability of a lost cause, and $\Pr[red|a_{-i}, F]$ the probability of being redundant:

$$\begin{aligned} Pr\left[piv|a_{-i}, F\right] &= \sum_{x=1}^{\infty}\left(\Pr\left[C_{-i} = x-1|a_{-i}\right] f(x)\right) \\ Pr\left[lost|a_{-i}, F\right] &= \sum_{x=1}^{\infty}\left(\Pr\left[C_{-i} < x-1|a_{-i}\right] f(x)\right) \\ Pr\left[red|a_{-i}, F\right] &= \sum_{x=1}^{\infty}\left(\Pr\left[C_{-i} > x-1|a_{-i}\right] f(x)\right). \end{aligned}$$

[6] To illustrate, a uniform pdf over $\{3,4\}$ is single-peaked, but it is not strictly quasi-concave because it is flat at $f(3)$ and $f(4)$.

Given a_{-i} and F, a player is willing to contribute if her expected payoff contributing exceeds that of not contributing:

$$
\begin{aligned}
& \Pr\left[lost|a_{-i}, F\right](-c) + \Pr\left[piv|a_{-i}, F\right](v-c) + \Pr\left[red|a_{-i}, F\right](v-c) \geq \Pr\left[red|a_{-i}, F\right] v \\
\Rightarrow \quad & \Pr\left[piv|a_{-i}, F\right] \geq \frac{c}{v}.
\end{aligned}
$$

It follows that for each i in a pure Nash equilibrium:

$$
a_i = \begin{cases} 0 & \text{if} \quad \Pr\left[piv|a_{-i}, F\right] < \frac{c}{v} \\ a' \in \{0,1\} & \text{if} \quad \Pr\left[piv|a_{-i}, F\right] = \frac{c}{v} \\ 1 & \text{if} \quad \Pr\left[piv|a_{-i}, F\right] > \frac{c}{v} \end{cases} \tag{1}
$$

Denote a pure equilibrium by C^*, and (abusing notation) let C^* also signify the number of contributions in that equilibrium. In a pure equilibrium C^* a contributing player believes with probability one that exactly C^*-1 others are contributing. Thus, that player is pivotal with probability $f(C^*)$. A non-contributing player is pivotal with probability $f(C^*+1)$. It follows that conditions for existence of an equilibrium C^* are:

$$
C^* = \begin{cases} 0 & \text{if} \quad f(1) \leq \frac{c}{v} \\ x \in \{1,..,n-1\} & \text{if} \quad f(x) \geq \frac{c}{v} \text{ and } f(x+1) \leq \frac{c}{v} \\ n & \text{if} \quad f(n) \geq \frac{c}{v} \end{cases} . \tag{2}
$$

Proposition 1 contains some preliminary conditions for uniqueness of equilibria. In it and throughout the rest of the paper, the *feasible mode* m is the mode of the distribution over $x \in \{1, ..., N\}$. More formally, $x \in N$ is the feasible mode where $f(x) > f(x')$ for all $x' \in N$, $x' \neq x$ ("$>$" by strict quasi-concavity).

Proposition 1: (Uniqueness of Pure Equilibria) *Fix N, c, v, and F.*

(a) The unique equilibrium is $C^* = 0$ iff $\frac{c}{v} > f(m)$. The unique equilibrium is $C^* = n$ iff $\frac{c}{v} < f(x)$ for all $x \in N$.

(b) If F is strictly quasi-concave, then any non-zero equilibrium has $C^* \geq m$.

(c) If F is strictly quasi-concave, then there is at most one non-zero equilibrium with $C^* > 0$. Furthermore, if there is more than one equilibrium then there are exactly two equilibria: the trivial equilibrium $C^* = 0$ and a non-zero equilibrium $C^* > 0$.

Proof: (a) Follows directly from the conditions in (2).

(b) By (2), any interior equilibrium $C^* > 0$ must have $f(C^*) \geq \frac{c}{v} \geq f(C^* + 1)$. By strict quasi-concavity, C^* must thus be to the right of m where the pdf is "downward sloping."

(c) Suppose two non-zero equilibria C^* and $C^{*\prime}$ s.t. $C^{*\prime} > C^* > 0$. From (2), it must be true that (i) $f(C^*) \geq \frac{c}{v}$, (ii) $f(C^* + 1) \leq \frac{c}{v}$, and (iii) $f(C^{*\prime}) \geq \frac{c}{v}$. But (i)-(iii) cannot be all be true since f is strictly quasi-concave. A contradiction.

Figure 1(a) illustrates the equilibria. In this figure, there is a trivial equilibrium $C^* = 0$ since $f(1) < \frac{c}{v}$. That is, if no one else contributes, then a single contribution is very unlikely to lead to provision, so no individual will contribute. The non-trivial equilibrium is $C^* = 3$. Each of the three contributors is willing to make her contribution given that two others are contributing since her probability of being pivotal is higher than $\frac{c}{v}$. Any non-contributor is not willing to contribute given that three others contribute since her probability of being pivotal is less than $\frac{c}{v}$. Also, notice that for a strictly quasi-concave distribution, a non-zero equilibrium must be to the right of the feasible mode $m = 3$ (on the downward-sloping side of the pdf). This fact follows from the features of the equilibrium described in (2): $\left(f(C^* + 1) \leq \frac{c}{v}\right)$ so that $f(C^*) \geq \frac{c}{v} > f(C^* + 1)$.

Hereafter, we will focus on the *non-trivial equilibrium* C^*. From Proposition 1(c), a zero-contribution equilibrium is non-trivial only if $\frac{c}{v}$ is higher than the feasible mode. Otherwise, the non-trivial equilibrium has contributions. As shown later, this non-trivial equilibrium is the Pareto-undominated equilibrium, although it can be inefficient.

I now state the two main theoretical propositions of this paper. The first of these will consider a threshold distribution that is totally feasible. Say that F is *totally feasible* when the public good can be provided with probability 1, i.e., $F(n) = 1$.

Proposition 2: (2nd-order Stochastic Dominance) *Fix N, c, and v, and consider two strictly quasi-concave threshold distributions F and $\widehat{F}$, $F \neq \widehat{F}$. Denote C^* and $\widehat{C}^*$ the respective non-trivial equilibria. If (i) F 2nd-order stochastically dominates $\widehat{F}$, (ii) F and $\widehat{F}$ have the same mean, and (iii) F and $\widehat{F}$ are both* *totally feasible, then there exists a scalar $k > 0$ such that $\widehat{C}^* \geq C^*$ if the cost-value ratio $\frac{c}{v} \leq k$. Furthermore, if it is also true that the feasible mode of F is strictly greater than the feasible mode of $\widehat{F}$, then there exists a second scalar $k' \geq k$ such that $C^* \geq \widehat{C}^*$ if the cost-value ratio $\frac{c}{v} > k'$.*

Proposition 3: (Single-Crossing Condition) *Fix N, c, and v, and consider two strictly quasi-concave threshold distributions F and $\widehat{F}$, $F \neq \widehat{F}$, with feasible modes m and $\widehat{m}$, respectively. Denote C^* and $\widehat{C}^*$ the respective non-trivial equilibria. If $f(m) > \widehat{f}(\widehat{m})$ and f and $\widehat{f}$ cross exactly once over $\{m, ..., n\}$, then there exists a scalar $k > 0$ such that $\widehat{C}^* \geq C^*$ if $\frac{c}{v} \leq k$, and $C^* \geq \widehat{C}^*$ if $\frac{c}{v} > k$.*

The proofs of Proposition 2 and 3 will follow directly from a more general result, Lemma 1, about games that differ only in their threshold distributions. In the rest of Section 3.1, I establish Lemma 1, after which Propositions 2 and 3 will be proven.

Because $C^* \geq m$, we can restrict our attention to that part of the threshold pdf that is between m and n. And we can go one step further when comparing the non-trivial equilibria of otherwise identical games with different threshold distributions. The following corollary to Proposition 1 states that when looking for the equilibrium with higher contributions of the two games, we can restrict our attention to that part of the two distributions that is between the feasible mode with higher mass and n. In other words, if $f(m) > \widehat{f}(\widehat{m})$, then we need only be concerned with the range $\{m, ..., n\}$. Denote $\widetilde{m}$ the feasible mode with higher mass.

Corollary 1: (Comparing Non-trivial Equilibria) *Fix N, c, and v, and consider two strictly quasi-concave threshold distributions F and $\widehat{F}$, $F \neq \widehat{F}$. Denote C^* and $\widehat{C}^*$ the respective non-trivial equilibria.*

(a) $\widehat{C}^* > C^*$ iff there exists some level of contributions $x \in \{C^* + 1, ..., n\}$, such that $\widehat{f}(x) \geq \frac{c}{v}$.

(b) If $\widehat{C}^* > C^*$ then $\widehat{C}^* \geq \widetilde{m}$.

With our attention now restricted to the right of the feasible mode with higher mass, we look more closely at the behavior of the two distributions from $\widetilde{m}$ to n. I will refer to this specific area as the *interior* $I = \{\widetilde{m}, ..., n\}$. One key condition of interest is when one of the pdf's has a higher interior-right tail, that is, one pdf is greater than the other pdf for all contribution levels from some number in the interior I to n. The "interior-right" means that we are looking at the right tail in this interior I. Another key condition is as analog for the interior-left, but this condition will also be defined by the height of the pdf's to the right of the interior-left. After formally defining these conditions, I will illustrate them graphically.

Interior Tails Conditions: *Consider two strictly quasi-concave distributions* F *and* $\widehat{F}$, $F \neq \widehat{F}$, *and the resulting interior* $I = \{\widetilde{m}, ..., n\}$.

(a) Say that $\widehat{f}$ *has a fatter interior-right tail than* f *if there exists an* $x \in I$, *such that* $\widehat{f}(x') \geq f(x')$ *for all* $x' \in \{x, ..., n\}$.

(b) Say that f *has a fatter interior-left tail than* $\widehat{f}$ *if there exists an* $x \in I$, *such that (i)* $f(x') \geq \widehat{f}(x')$ *for all* $x' \in \{\widetilde{m}, ...x\}$, *and (ii) if* $\widehat{m} > m$ *with* $\widehat{f}(\widehat{m}) > f(\widehat{m})$, *then* m $f(x') > \widehat{f}(\widehat{m})$ *for all* $x' \in \{\widetilde{m}, ..., x\}$.

Figure 2(a) illustrates these conditions with smooth pdf's drawn for clarity. It shows the case where both a fatter interior-right tail I_R and a fatter interior-left tail I_L exist. Notice that these tails do not necessarily meet. Figure 2(b) shows that we cannot distinguish I_R from I_L when one pdf is always above the other in I. Figures 2(c)-(d) show that the tails meet when the pdfs cross once in the interior. The reason for condition (ii) in a fatter interior-left tail is that we want to know when the non-trivial equilibrium C^* will be in that interior-left tail *and* when $C^* \geq \widehat{C}^*$. This idea is illustrated on Figure 2(b). Notice that if $\frac{c}{v} = k_1$, then $\widehat{C}^*$ is higher than $C^* = x_1$ even though $f(x_1) > \widehat{f}(x_1)$. This is because x_1 is to the left of the feasible mode of $\widehat{F}$.

We can use these figures to demonstrate the two main propositions of the paper and bring us closer to Lemma 1. Notice that the k and k' in Figure 2(a) satisfy the k and k' in Proposition 2. F 2nd-order stochastically dominates $\widehat{F}$, and F has a higher feasible mode. We see that if $\frac{c}{v} \leq k$ then $\widehat{C}^* \geq C^*$, whereas if $\frac{c}{v} > k'$ then $C^* \geq \widehat{C}^*$.

Lemma 1: (Fatter Interior Tails and Pure Equilibria) *Consider two games that are identical except for their strictly quasi-concave threshold distributions F and $\widehat{F}$, $F \neq \widehat{F}$. Denote C^* and $\widehat{C}^*$ the respective non-trivial equilibria.*

(a) If $\widehat{f}$ has a fatter interior-right tail than f, then there exists a scalar k, $0 < k < 1$, such that $\widehat{C}^* \geq C^*$ if the cost-value ratio $\frac{c}{v} \leq k$.

(b) If f has a fatter interior-left tail than $\widehat{f}$, then there exists a scalar k', $0 < k' < 1$, such that $C^* \geq \widehat{C}^*$ if the cost-value ratio $\frac{c}{v} > k'$.

Proof: (a) Suppose the contrary, that $\widehat{f}$ has the fatter interior-right tail but $C^* > \widehat{C}^*$ for some $\frac{c}{v} \leq k$. Choose $k = f(\widehat{m}_R)$, where $\widehat{m}_R$ is the mode of $\widehat{f}$ over the fatter interior-right tail $\{x, ..., n\}$. It follows that $\frac{c}{v} \leq k$ implies that $C^* \geq x$, where x is the beginning of the fatter interior-right tail. For $C^* > \widehat{C}^*$, it must be that $f(x') \geq \frac{c}{v} > \widehat{f}(x')$ for some x', $x \leq x' \leq n$, but this contradicts the fact that $\widehat{f}$ has a fatter interior-right tail. Similar logic will show that any $k \in \left[0, \widehat{f}(\widehat{m}_R)\right]$ will satisfy the claim.

(b) Follows from logic similar to that used in part (a).

Lemma 1(a) says that contributions will be higher under the distribution with the fatter interior-right tail if $\frac{c}{v}$ is sufficiently small, Lemma 1(b) says that contributions will be higher under the distribution with fatter interior-left tail if $\frac{c}{v}$ is sufficiently large. The reason is that the fatter tail implies a higher probability of being pivotal at the contribution levels in the tail. We can now prove Propositions 2 and 3.

Proof of Proposition 2: F 2nd-order stochastically dominates $\widehat{F}$ implies that $\sum_{x=0}^{x'} \widehat{F}(x) \geq \sum_{x=0}^{x'} F(x)$ for all $x' \in N$. Total feasibility and same means together imply that $\sum_{x=0}^{n} \widehat{F}(x) = \sum_{x=0}^{n} F(x)$ (see Laffont 1989). Subtracting the first condition from the second condition yields

$$\begin{aligned} \sum_{x=0}^{n} \widehat{F}(x) - \sum_{x=0}^{x'} \widehat{F}(x) &\leq \sum_{x=0}^{n} F(x) - \sum_{x=0}^{x'} F(x) \\ \sum_{x=x'+1}^{n} \widehat{F}(x) &\leq \sum_{x=x'+1}^{n} F(x). \end{aligned}$$

This last equation says that, starting from n and moving to the left on the graph of the cdf's, when F and $\widehat{F}$ first separate, $\widehat{F}$ must be below F. This implies that $\widehat{f}$ must have a fatter interior-right tail than f. (Notice that if F and $\widehat{F}$ do not separate in interior I, then $\widehat{f}$ and f have identical interiors, which means that $\widehat{f}$ has a fatter interior-right tail by the weakness.) Since $\widehat{f}$ has a fatter interior-right tail, invoking Lemma 1 establishes that there exists a k that satisfies the first claim in Proposition 2.

If the feasible mode of f has mass strictly greater than the feasible mode of $\widehat{f}$ then it follows that f has a fatter interior-left tail. Invoke Lemma 1 to establish that there exists a k' as in the second claim in Proposition 2.

The intuition for Proposition 2 is straightforward. Spreading the distribution pushes probability mass to the right part of the tail, and the total feasibility restriction means that this mass will stay in the feasible region. With more mass in the interior-right tail, the probability of being pivotal is higher at the high levels of x. Alone, this is not enough to ensure that contributions will be higher in the game with the spread probability. If the cost-value ratio $\frac{c}{v}$ is too high, then the mass increase on the right will not be enough, and there might be a drop in contributions. This is seen in Figure 2(a) when $\frac{c}{v} > k'$. If the cost is low enough (below k) then contributions are higher.

Notice that total feasibility is sufficient but not necessary. What is necessary in this case of a mean-preserving spread is that enough mass is spread to the interior-right. In other words, all we need for Proposition 2 is a fatter interior-right tail (Lemma 1). Proposition 3 demonstrates this point (because it does not assume total feasibility) while making another claim about an implication of the single-crossing property.

Proof of Proposition 3: The claim assumes that $I = \{m, ..., n\}$ (i.e., $\widetilde{m} = m$). Suppose the pdf's cross at $x \in \{m, ..., n\}$, so that $f(x') \geq \widehat{f}(x')$ for all $m \leq x' < x$, and $f(x'') \leq f(x'')$ for all $x \leq x'' \leq n$. It follows then that $\widehat{f}$ has a fatter interior-right tail and f has a fatter interior-left tail. It remains to show that $k = k'$ in Lemma 1.

If $f(\widehat{m}) \geq \widehat{f}(\widehat{m})$ then by strict quasi-concavity, f has a fatter interior-left tail from m to $x+1$ and $\widehat{f}$ has a fatter interior-right tail from x to n. These two tails meet each other, so $k = k'$ in Lemma 1, thus satisfying the claim. Now suppose that $f(\widehat{m}) < \widehat{f}(\widehat{m})$ (akin to Figure 3.2(b)). Set $k = \widehat{f}(\widehat{m})$, and then find where f crosses k. For any $\frac{c}{v} \leq k$, we satisfy Lemma 1(a), and for any $\frac{c}{v} > k$, we satisfy Lemma 1(b). Thus $k = k'$ in Lemma 1.

Proposition 3 applies for a wide variety of threshold distributions. For example, many monotone mean-preserving spreads will meet this single-crossing condition, such as shown in Figure 2(d). Also, the class of uniform threshold distributions meets this single-crossing condition. I take advantage of this last fact in the experiment I conducted (see Section 4).

3.2 Mixed Equilibria

Similar logic is used in examining the mixed equilibria, but there is one important difference. While the results for pure equilibria come from looking at fatter interior tails of the probability distributions, the results for the mixed equilibria come from looking at fatter interior tails of *transformations* of the probability distributions.

For reasons given below, when looking at mixed equilibria, we can restrict our attention to symmetric equilibria. With α_i the probability that player i contributes, we now let $\alpha = \alpha_i = \alpha_j$, for all $i, j \in N$, be the rate at which every player mixes. From (1), we see that the conditions for a symmetric equilibrium are

$$\alpha_i^* = \begin{cases} 0 & \text{if} \quad \Pr[piv|\alpha^* = 0, F] < \frac{c}{v} \\ \alpha' \in [0,1] & \text{if} \quad \Pr[piv|\alpha^* = \alpha', F] = \frac{c}{v} \\ 1 & \text{if} \quad \Pr[piv|\alpha^* = 1, F] > \frac{c}{v} \end{cases} . \tag{3}$$

The transformation of the probability distribution of interest is what I will call the $\Pr[piv|F]$-curve. This curve maps the probability player i is pivotal given that all others are mixing at rate $\alpha \in [0,1]$. Figure 1(b) illustrates this $\Pr[piv|F]$-curve for the pdf in Figure 1(a). This curve is derived as follows:

$$\Pr[piv|\alpha, F] = \sum_{x=1}^{n} \binom{n-1}{x-1} \alpha^{x-1} (1-\alpha)^{n-x} f(x).$$

With $N = 5$, there are three symmetric equilibria in Figure 1(b): $\alpha^* = 0$, $\alpha^* = 0.32$, and $\alpha^* = 0.91$. From the conditions in (3), it follows that symmetric equilibria can only occur at three places on the graph: at the origin if the $\Pr[piv|F]$-curve is less than $\frac{c}{v}$ at $\alpha = 0$, at a place where the $\Pr[piv|F]$-curve intersects the $\frac{c}{v}$-line, and at $\alpha = 1$ if the $\Pr[piv|F]$-curve crosses it above $\frac{c}{v}$. This last possibility would happen in Figure 1(b) if $\frac{c}{v} \leq 0.15$.

Equilibria at 0 and 1 have a nice stability property: an ε increase in α from 0 would drive contributions back down to zero, and an ε decrease in α from 1 would drive contributions back to one. Strictly mixing equilibria only share this property if the slope of the $\Pr[piv|F]$-curve is downward sloping where it crosses the $\frac{c}{v}$-line. In Figure 1(b), the equilibrium at 0.32 is not stable, but the one at 0.91 is stable. The stable symmetric equilibria have qualitative properties similar to the pure equilibria: they occur where the distribution (or its $\Pr[piv|F]$-curve transformation) crosses the $\frac{c}{v}$ line from above. We take advantage of this fact in the propositions and corollaries for symmetric equilibria.

This stability notion coincides with the concept of evolutionarily stable strategies (ESS) (see Gintis (2000)): i is at least as better off playing α than playing the perturbed strategy given that the others play α, and if i is indifferent to playing the perturbed strategy given the other play α, then i is strictly better off playing α than playing the perturbed strategy when all others play the perturbed strategy. I now use this stability concept to state Proposition 1A, which is the $\Pr[piv|F]$-curve analog to Proposition 1, with the addition of part (0).

Proposition 1A: (Uniqueness of Symmetric Equilibria) *Fix N, c, v, and F.*

(0) If there is a stable equilibrium in which at least two players i and j are strictly mixing $\alpha_i^*, \alpha_j^* \in (0,1)$, then it must be that $\alpha_i^* = \alpha_j^*$ (generically).

(a) The unique equilibrium is $\alpha = 0$ iff $\frac{c}{v}$ is strictly greater than the maximum of the $\Pr[piv|F]$-curve. The unique equilibrium is $\alpha^* = 1$ iff $\frac{c}{v}$ is strictly less than $\Pr[piv|F]$-curve for all $\alpha^* \in [0,1]$.

(b) If the $\Pr[piv|F]$-curve is strictly quasi-concave, then any stable equilibrium with $\alpha^* > 0$ has α^* (weakly) to the right of the mode of the $\Pr[piv|F]$-curve.

(c) If the $\Pr[piv|F]$*-curve is strictly quasi-concave, then there is at most one stable equilibrium with* $\alpha^* > 0$*. Furthermore, if there is more than one stable equilibrium then there are exactly two stable equilibria: the trivial equilibrium* $\alpha^* = 0$ *and a non-trivial equilibrium* $\alpha^* > 0$*.*

Proof: (0) Suppose an equilibrium with i and j both strictly mixing and $\alpha_i^* < \alpha_j^*$. Equilibrium implies that each must have a probability of being pivotal equal to $\frac{c}{v}$ by (3). Since j is mixing at a higher rate than i, i's expected number of contributors other than himself must be higher than j's expected number of contributors other than himself. However, this means that i and j do not have equal probabilities of being pivotal (generically), which means that both cannot have probabilities of being pivotal equal to $\frac{c}{v}$ which is a contradiction.

(a)-(c) Follows from logic used in proving Proposition 1.

I focus now on non-trivial equilibria that are stable and symmetric, and part (0) provides justification. Any strict mixers must mix at the same rate, so if a mixed equilibrium is asymmetric, the asymmetry is in who mixes and not the rate at which they mix. In fact, the mixing rate in one of these asymmetric equilibria is the mixing rate in a symmetric equilibrium of a transformed game.[7] As a result, we are examining the main strategic aspects[8] of all mixed equilibria when considering symmetric equilibria. We can justify looking at stable equilibria, too. First, the ESS concept has nice stability properties that suggest that such strategies are more likely to be observed. Second, ESS can arise out of many dynamic processes which again suggests they are more likely to be observed.[9] Third,

[7] More precisely, if N_{mix} is the set of mixers and N_1 is the set of pure contributors, then the rate at which the mixers is mixing is equal to the mixing rate in game G' with $f'(x) = f(x + |N_1|)$ for all $x > 0$.

[8] The only aspect missing is the possibility of a different number of strict mixers.

[9] Here are two examples of dynamic processes that lead to an ESS being reached. The first example is a restricted best-response dynamic process. Suppose in period t, each player chooses a best response to the strategies of the previous period with the restriction that $BR_{i,t} \in [a_{i,t-1} - \delta, a_{i,t-1} + \delta] \subset [0,1]$. In other words, each player is restricted to making only small deviations from his previous period's strategy. Without this restriction, players would jump back and forth between contributing and not contributing, and no mixed equilibrium would be reached. In these dynamics, only ESS will be reached if the system starts out of equilibrium. The second example of a dynamic process is based on the interpretation of mixed strategies in terms of large population, random interaction models in which players only play pure strategies. Consider a large population of players that is randomly divided into n-sized groups over a long period of time. If

symmetric mixed ESS will exhibit comparative static properties that are qualitatively similar to the asymmetric pure equilibria thereby giving added justification to the comparative static predictions of these equilibria.

The analog to the non-trivial pure equilibrium C^* is the non-trivial stable and symmetric equilibrium α^*. Lemma 1 can be restated as Lemma 1A in terms of the fatter interior tails of the $\Pr[piv|F]$-curves.

> **Lemma 1A:** (Fatter Interior Tails and Symmetric Equilibria) *Consider two games identical except for their threshold distributions F and $\widehat{F}$, $F \neq \widehat{F}$. Denote α^* and $\widehat{\alpha}^*$ the respective symmetric and stable non-trivial equilibria.*
>
> *(a) If the $\Pr\left[piv|\widehat{F}\right]$-curve has a fatter interior-right tail than the $\Pr\left[piv|F\right]$-curve, then there exists a scalar k, $0 < k < 1$, such that $\widehat{\alpha}^* \geq \alpha^*$ if the cost-value ratio $\frac{c}{v} \leq k$.*
>
> *(b) If the $\Pr\left[piv|F\right]$-curve has a fatter interior-left tail than the $\Pr\left[piv|\widehat{F}\right]$-curve, then there exists a scalar k', $0 < k' < 1$, such that $\alpha^* \geq \widehat{\alpha}^*$ if the cost-value ratio $\frac{c}{v} > k'$.*

A 2nd-order stochastic dominance relationship between threshold distributions F and $\widehat{F}$ does not necessarily imply a 2nd-order stochastic dominance between the $\Pr[piv|F]$- and $\Pr\left[piv|\widehat{F}\right]$-curves. However, a dominance relationship between F and $\widehat{F}$ will generally imply that the $\Pr\left[piv|\widehat{F}\right]$-curve has a fatter interior-right tail, and it is the fatter interior-right tail that really matters. This means that there will generally exist the k that yields $\widehat{\alpha}^* \geq \alpha^*$ whenever $\frac{c}{v} \leq k$, which is the symmetric equilibrium analog to Proposition 2.

Similarly, when m is strictly higher than $\widehat{m}$, the mode of the $\Pr[piv|F]$-curve will often have a higher mass than the mode of the $\Pr\left[piv|\widehat{F}\right]$-curve. This will result in a fatter

players that receive higher payoffs reproduce at faster rates, convergence will occur to a population where a fraction of the population that contributes towards the public good will be equal to the mixed strategy equilibrium. Each of these two examples have dynamics that will lead to convergence to identical equilibria, either mixed in the first model or pure but in the mixed proportions in the second model. Equilibria that are not ESS cannot be sustained with pertubations under any evolutionary model. We therefore expect to see actual behavior fall more in line with these ESS.

interior-left tail for the $\Pr[piv|F]$-curve, which is sufficient for the existence of the k' that yields $\alpha^* \geq \widehat{\alpha}^*$ whenever $\frac{c}{v} > k'$, which completes the symmetric analog to Proposition 2.

3.3 Efficiency

We are interested in the efficiency of equilibria when there is threshold uncertainty and also in the comparative efficiency of equilibria under different threshold distributions. The welfare criterion used here is the sum of expected utilities: $W(C) = nF(C)v - Cc$.

Proposition 4: (Efficiency) *Fix N, c, v, and F. Assume F is strictly quasi-concave.*

(a) The non-trivial pure equilibrium C^* is Pareto-undominated in the class of pure equilibria, and C^* is inefficient if $C^* < n$ and $\frac{c}{vn} < f(C^*+1) < \frac{c}{v}$.

(b) The symmetric and stable non-trivial equilibrium α^* is generically inefficient, but it can yield higher expected welfare than the non-trivial pure equilibrium C^*.

Proof: (a) From Proposition 1(c), we know that if there is more than one pure equilibrium, then one is $C^{*\prime} = 0$ while the other is, say, $C^* > 0$. The expected welfare of $C^{*\prime} = 0$ is 0. Since it must be true that $f(C^*) \geq \frac{c}{v}$, it must also be true that $F(C^*) \geq \frac{c}{v}$. This implies that the expected welfare of C^* is

$$nF(C^*)v - C^*c \geq n\frac{c}{v}v - C^*c = (n - C^*)c,$$

which is weakly greater than 0. Thus, C^* is Pareto undominated.

Consider the second claim in (a). Let $C^* \in \{1, ..., n-1\}$. By Proposition 1, C^* is to the right of the feasible mode. By strict quasi-concavity, $f(C^*) \geq \frac{c}{v} > f(C^*+1) \geq f(C^*+k)$ for all $1 < k \leq n - C^*$. This means that the largest marginal welfare gain to be had by an increase in one contribution is from C^* to C^*+1. Welfare is higher under C^*+1 when $W(C^*+1) > W(C^*)$. Doing the algebra shows this to be equivalent to $f(C^*+1) > \frac{c}{vn}$. It follows that the C^* is inefficient when $\frac{c}{vn} < f(C^*+1) < \frac{c}{v}$.

(b) That mixed equilibria are generically inefficient is trivial. That the symmetric equilibrium can yield higher expected welfare than the pure equilibrium is illustrated by an example. Suppose $n = 5$, $f(1) = 0.54$, $f(2) = 0.13$, $f(3) = 0.12$, $f(4) = 0.11$, $f(5) = 0.10$, and $\frac{c}{v} = 0.14$. Then we can find that $C^* = 1$, $\alpha^* \simeq 0.5$, $W(C^*) = 18.29$, and $W(\alpha^*) \simeq 23$.

As is common in public good games, inefficiencies arise because the marginal gain to an individual from contributing is different from the marginal social gain from that same contribution. This difference comes from the welfare function accounting for all players' marginal gain's instead of just one individual's marginal gain. This inefficiency does not arise when $f(C^* + 1) < \frac{c}{v}$, $C^* = n$, or when $F(C^*) = 1$. Notice that this implies that the non-trivial equilibrium C^* is efficient when the threshold is known with certainty—a fact already established by Palfrey and Rosenthal (1984). Their result is thus a special case of the more general result in Proposition 4(a).

The symmetric equilibrium can have higher expected welfare when the pdf has a tail to the right of C^* that is close to but under $\frac{c}{v}$. This is the case in the example in part (b)'s proof. The symmetric equilibrium has higher welfare because expected contributions are higher, and the higher contributions leads to a higher probability of provision that more than offsets the decline in welfare due to greater total contribution cost.

Because contributions can increase due to an increase in uncertainty, welfare can be higher under an increase in uncertainty. Again, suppose that the initial distribution has a right tail above C^* that is below $\frac{c}{v}$ but above $\frac{c}{vn}$ from $C^* + 1$ to n or close to n. A widening of uncertainty that drives up the right tail will increase contributions, and if the increase in the probability of provision is sufficient then there will be an increase in expected welfare.

3.4 Other Considerations

General Threshold Distributions. I have worked out the analogs to the main claims for when the threshold distributions are not restricted by strict quasi-concavity. The added complication is the non-uniqueness of non-trivial equilibrium. One way around this complication is to look at the equilibrium with the highest level of expected contributions. Doing so allows us to do the same analysis as before on this high-contribution equilibrium. For pure equilibria, this high-contribution equilibrium is the Pareto-undominated equilibrium, and Lemma 1 and Propositions 2 and 3 can be restated exactly word for word substituting only "high-contribution equilibrium" in place of "non-trivial equilibrium." The analysis will also be similar for symmetric equilibria.

Continuous Contributions. Nitzan and Romano (1990) allow individuals to make continuous contributions. These continuous contributions can be likened to monetary contributions, whereas binary contributions can be likened to participation decisions. Leaving the binary case means we must consider the underlying threshold distribution. Proposition 5 assumes that the underlying threshold distribution is a continuous and strictly quasi-concave threshold distribution function H from which the discrete transformations F and F' are derived so as to assign mass over A_i and A'_i, respectively.

> **Proposition 5:** (Continuous Contributions, partly from Nitzan and Romano (1990)) *Consider two games identical except for their contribution sets A_i and A'_i. Assume that the threshold distribution function H is continuous. Suppose binary contributions $A_i = \{0, 1\}$ for all i in the first game, and assume continuous contributions $A'_i = [0, 1]$ for all i in the second game. Then expected welfare is always (weakly) higher in the second game with continuous contributions.*

I sketch the proof using Proposition 4 and a result from Nitzan and Romano (1990). They show that in continuous contribution games we need to consider the maximum number of possible contributions. I restrict players to contribute at most 1, thus making n the maximum contributions. In their notation, H is continuous over $[a, b]$, $0 < a < n$. If $b \leq n$ with continuous contribution game, then $C^* = b$,[10] the public good is provided with probability 1, and the equilibrium is efficient. If $b < n$, then $C^* = n$ and the public good is provided with probability strictly less than 1. Proposition 4 says that things change

[10] Here C^* is the total contribution but not necessarily the number of contributors because C^* is not necessarily an integer.

with binary contributions. First, with total feasibility ($n \geq b$) then the equilibrium is only efficient if $f(b) \geq \frac{c}{v}$. Second, if not totally feasible, then it is only efficient if $f(n) \geq \frac{c}{v}$.

Essentially, the equilibrium under continuous contributions will always be efficient, but the equilibrium is not always efficient under binary contributions. Consider a proposed equilibrium $C^* < b$. In the binary case, a non-contributor considers if $f(C^* + 1)$ is greater than $\frac{c}{v}$. Now in the continuous case, if the player considers a $\frac{1}{2}$ contribution, then he compares $f\left(C^* + \frac{1}{2}\right)$ with $\frac{c}{2v}$ in his decision rule. More generally, it can be shown that for a $\frac{1}{2m}$ contribution, the player's decision rule will compare $f\left(C^* + \frac{1}{2m}\right)$ with $\frac{c}{2mv}$. As $m \to \infty$, the $\frac{c}{2mv}$-line converges to 0 and $f\left(C^* + \frac{1}{2m}\right)$ converges to $f(C^*)$. In the limit, the player he will contribute an ε amount whenever $f(C^*) > 0$. Thus, contributions will cover the whole feasible domain of the threshold distribution. For symmetric equilibria, similar reasoning will show that with continuous contributions, $\alpha^* = \frac{b}{n}$ when $n \geq b$.

This logic implies that wider threshold uncertainty can only decrease efficiency for the continuous contribution case (when b increases past n), while wider uncertainty can increase efficiency under binary contributions (even if b goes past n). While this is a strikingly different result, the underlying behavior and analysis in each case is the same. The difference lies in the fact that we do not always have complete provision of the public good in the binary case due to the $\frac{c}{v}$-line above the horizontal axis.

Risk Aversion. If players are risk averse then the free-rider effect (the worry about donating a redundant contribution) diminishes while the lost-cause effect (the worry about contributing to a lost-cause) amplifies. A qualitative result similar to Lemma 1 will hold, but there is an important difference. The decision rule (1) will not compare $\Pr[piv|a_{-i}, F]$ with $\frac{c}{v}$. Instead of drawing a horizontal $\frac{c}{v}$-line, there will be a curve that varies by contribution level. On the graph of the pdf, this curve will be decreasing over the domain of contribution levels with $f(x) > 0$, and its slope and shape will depend on the size of the risk aversion. Under extreme amounts of risk aversion, the slope becomes more negative and the whole curve shifts up. With a change in uncertainty from F to $\widehat{F}$, the curve will also change. For the analog to Lemma 1(a), our definition of the fatter interior-right tail will have to consider not just the comparison pdf's but also the comparison of these curves.

Sequential Contributions. Since there is no private information in this game, there are not the normal informational issues involved in comparing simultaneous and sequential equilibria. Sequential moves only allow players to condition on observed behavior. This observation will matter when comparing a mixed equilibrium to a sequential equilibrium, but any pure equilibrium of the simultaneous game is an equilibrium of any sequential move game.[11] The main results from Section 3.1 will thus still apply in the sequential move game. Hence, the focus on simultaneous contributions in this paper is not missing other important strategic issues (other than timing) that would arise in a sequential move game.

4 Experiment

Laboratory experiments provide the ideal setting to conduct a preliminary empirical test of the model's theoretical predictions—that contributions will be higher under wider threshold uncertainty if the public good is of sufficiently high value—since the experimenter can control both the level of threshold uncertainty and also changes in that uncertainty. This section presents the main findings from an experiment I conducted at the California Social Science Experimental Laboratory. All subjects are drawn from the student population of the University of California, Los Angeles.[12]

4.1 Experiment Design

Each experiment session consisted of 4 practice rounds and 30 real rounds,[13] and each session had either 25 or 30 students. All decisions were made over a computer network in a computer currency called "tokens." Subjects amassed tokens depending on the decisions and the factors determined by the computer. At the end of the session, subjects were paid U.S. dollars according to a pre-announced token/dollar exchange rate.

[11] Dekel and Piccione (2000) have a similar finding for voting games in symmetric binary elections.
[12] The dialogue from the instructional period is available from the author upon request.
[13] The exception is the 8/21 session which ended after 26 rounds.

In each round, the computer randomly and anonymously assigns the subjects into groups of five, and each subject is given one computer token. Each subject's computer then displays the public good value and the threshold distribution. Subjects are told that the threshold distribution is a range $T = \{\underline{t}, ..., \bar{t}\}$ from which the computer will randomly and uniformly select the true threshold. Subjects are told that the "threshold-met value" and "threshold range" are the same for all individuals and groups in the room.

Before deciding whether to keep (do not contribute) or spend (contribute) the one given token, each subject is asked to assign percentage probabilities to what the others in his or her group will do.[14] Since five subjects are in each group, a subject assigns probabilities to the following five events: exactly 0 others in the group spend, exactly 1 other spends, exactly 2 others spend, exactly 3 others spend, and exactly 4 others spend. Once the assigned percentages add up to 100 percent and the subject confirms the entry, the subject then makes the decision to keep or spend the one token. Tokens not spent in the current round cannot be spent in later rounds. Subjects are not allowed to communicate with any other subjects in the room during the practice or real rounds.

A subject's payment for a given round has two parts. The first payment is based on the accuracy of the reported beliefs, which is derived using a proper scoring rule. The exact formula is

$$\frac{v}{2}\left(b_{it}\left(actual_t\right) - \frac{1}{2}\left(b_{it}\left(0\right)^2 + b_{it}\left(1\right) + ... + b_{it}\left(4\right)^2\right)\right) + \frac{v}{4},$$

where $b_{it}\left(e\right)$, $e = 0, ..., 4$, is the percent assigned by i to the event that e others spend, and $b_{it}\left(actual_t\right)$ is the percent assigned to that x that actually occurs. The highest this payment can be in a given round is $\frac{v}{2}$, and the lowest is 0. The second payment in a given round is that resulting from the contribution decisions. Letting C be the total realized contributions in the group, this payment is:

$$\begin{array}{cl} v & \text{if} \quad C \geq t \text{ and } a_i = 0 \\ v+1 & \text{if} \quad C \geq t \text{ and } a_i = 1 \\ 1 & \text{if} \quad C < t \text{ and } a_i = 0 \\ 0 & \text{if} \quad C < t \text{ and } a_i = 1. \end{array}$$

[14]See Nyarko and Schotter (2000) for discussion of the validity of using elicited beliefs.

I consider five different value-threshold range (v, T) combinations:

$$(3, \{3\}), (3, \{1, 2, 3, 4, 5\})$$
$$(6, \{3\}), (6, \{2, 3, 4\}), (6, \{1, 2, 3, 4, 5\}),$$

in a variety of different treatments. While v was held fixed in each session, T varied in some sessions. Whenever the range was changed, it was fixed for the first 15 rounds, then changed to another range, which was then held fixed for the rest of the session. Table 1(a) lists the expected payoff maximization mixed equilibrium contribution levels under the different (v, T)-combinations. It also lists the qualitative predictions: contributions should be higher under $(3, \{3\})$ than $(3, \{1, 2, 3, 4, 5\})$, and they should be successively higher under $(6, \{3\})$, $(6, \{2, 3, 4\})$, and $(6, \{1, 2, 3, 4, 5\})$. Table 1(b) lists the different treatments and the number of sessions per treatment.

This design is the correct design to test my theoretical predictions for a number of reasons. (1) This basic set-up, including $n = 5$ and uniform threshold range, matches that used in the previous experimental studies of threshold uncertainty mentioned earlier. This establishes continuity with the other studies. (2) The uniform threshold range is the best way to model the threshold distribution since subjects understand a uniform distribution. The uniform threshold range also implies single-crossing for both pure and symmetric equilibria, and this single-crossing implies nice qualitative predictions of contribution movements with changes in uncertainty (Proposition 3). (3) The chosen parameters profiles will allow for high and low v and for high and low uncertainty. Data for all these scenarios are needed to compare with the predictions. (4) Eliciting beliefs will allow for more direct testing of the underlying behavior of the subjects. Providing incentives to report true beliefs adds credibility to the beliefs data. (5) Group sizes are held constant to remove the effects of group sizes on contribution levels. (6) No information on others' decisions or payments is given and all decisions are made privately to remove social pressures or social comparisons that might affect behavior. (7) The maximum payment for beliefs is half as much as the highest payment from the keep/spend decision. This should remove the motive for players to play a game that maximizes the beliefs payment.[15]

4.2 Results

Findings 1-4 summarize the main results.

> **Finding 1:** *The predictions concerning contribution levels under different* (v, T)*-combinations are moderately verified.*

The key phrase is "moderately verified." Table 2(a) lists the contribution levels by (v, T)-combination by all rounds and for rounds 8 and higher. The quantitative contribution levels differ substantially from the mixed equilibrium contribution levels in all case. However, when $v = 3$, contributions are higher under $T = \{3\}$ than under $T = \{1, 2, 3, 4, 5\}$ as qualitatively predicted by the model. In the later rounds where the predictions may be more likely to be verified (e.g., due to convergence to an equilibrium), contributions are over 7 percent higher. This difference is only weakly significant (a test of equal means gives a test statistic[16] of 1.50, not shown) because of the small sample size. When $v = 6$, contribution levels are higher under wider uncertainty as predicted, although the differences in levels are small. The difference between contributions under $\{3\}$ and $\{1, 2, 3, 4, 5\}$ and between $\{2, 3, 4\}$ and $\{1, 2, 3, 4, 5\}$ are weakly to moderately significant (test statistics -1.92 and -1.72, respectively). Contributions under $\{3\}$ and $\{2, 3, 4\}$ do not statistically differ.

Table 2(b) further breaks down contribution levels in the later rounds by session. Contribution means vary widely across sessions—even when under the same (v, T)-combination. Of the 7 sessions with multiple threshold ranges, contribution levels differ in the predicted

[15] Note that careful wording was used during the experiment. Words like "game," "contribute," "win," "lose," "reward," and "punishment" were not used since such words potentially carry subtle meanings that can affect behavior. Instead, words and phrases such as "decision making environment," "keep," "spend," and "payment" were used.

[16] The t-statistic for testing the equality of means two means p_x and p_y is

$$Z = \frac{\widehat{p}_x - \widehat{p}_y}{\sqrt{p_0 \left(1 - p_0\right) \left(\frac{n_x + n_y}{n_x n_y}\right)}},$$

where $p_0 = \frac{p_x n_x + p_y n_y}{n_x + n_y}$ (Newbold 1995, 360).

manner in 4 and differ opposite of the predicted manner in 3.[17] In the sessions where the range never changed, we again both match and do not match the predictions. As predicted, contributions under $\{1,2,3,4,5\}$ sessions are always higher than under $\{3\}$ and $\{2,3,4\}$, but contributions in the single $\{3\}$-session are slightly higher than under the three $\{2,3,4\}$-sessions.

In short, aggregated contribution levels differ qualitatively as expected in some cases but not all. But this verification is moderate at best. Two questions follow: why are the predictions verified to any degree, and why are they not verified to a larger degree?

I explore these questions using the data on subjects' elicited beliefs. I am not the first to use such data in the public good setting (see Offerman (1996)), however, this is this first use of beliefs data in the context of threshold uncertainty. Nyarko and Schotter (2000) use experimental evidence to show that elicited beliefs data are the best proxies for subjects' true beliefs. Finding 2 provides additional justification for the use of my particular data.

Finding 2: *The reported beliefs move in manners consistent with beliefs-updating.*

Let $\overline{b_{it}} = \sum_{e=0}^{n-1} eb_{it}(e)$ be the mean of i's reported beliefs in period t. Although subjects are randomly matched in each round, it is likely that subjects use contribution levels of the prior rounds to help predict what current group members will contribute. In this case, $\overline{b_{it}}$ will be closer to what happened in $t-1$ than $\overline{b_{it-1}}$. It will also be true that the probability assigned in $t-1$ to the event that actually occurred in $t-1$ will be higher than the probability assigned in $t-1$ (e.g., if i assigned 30 percent to 3 others spending in $t-1$ and 3 others spent in $t-1$ then i should assign 30 percent or more to 3 others spending in t).

Table 3(a) lists how frequently beliefs moved in these two manners for each session. The first round of a particular parameter profile is not included in the calculation of this percentage. Beliefs averages moved as expected between 72 percent and 79 percent of the time across the sessions, and 75 percent overall. Subjects (weakly) increased the probability

[17] In practice, the sessions within which the threshold range changes are the better ones to look at for testing my hypothesis because, as Camerer (1995) explains, the presence of individual, group, or session effects makes comparison across sessions more problematic.

placed on the last period's event between 78 percent and 85 percent of the time, and about 82 percent overall.

Table 3(b) presents estimates from regressions of $\overline{b_{it}}$ on different control variables. OLS regressions suffer from two potential problems. First, the regressions are double censored in that $\overline{b_{it}}$ must be between 0 and 4. Examination of the data reveals that of the 10,990 lagged observations only 132 observations had $\overline{b_{it}} = 4$, 6 had $\overline{b_{it}} = 0$, and one had incorrectly imputed values for $\overline{b_{it}}$. Removing these 139 observations (less than 1.5 percent of the data) leaves 10,851 observations, and regressions on these data should not suffer from inconsistencies that could result from the censoring. The first estimates listed are from an OLS regression of $\overline{b_{it}}$ on a constant, $\overline{b_{it-1}}$, and $\left(actual_{t-1} - \overline{b_{it-1}}\right)$ using this reduced set of observations. All estimates have the expected signs and are highly significant.

A second problem is that OLS does not account for possible autocorrelation, and the standard Durbin-Watson test indicates the presence of negative autocorrelation, as evidenced by a test statistic significantly different than 2. Autocorrelation is detected even though this statistic should be biased towards 2 because of the lagged dependent variable.[18] Table 3(b) presents results from two different 1st-degree autoregressions. The first AR(1) gives results similar to the OLS results. The second AR(1) includes more control variables that capture how beliefs-updating might differ in later rounds. The R^2 values over 60 percent indicate that a significant amount of the mean beliefs can be explained by the regressors used.

Finding 2 suggests that reported beliefs reflect the subjects' true beliefs. I also note that the computer interface did not list the subject's beliefs reports from prior rounds (it only lists contribution decisions and payments) when subjects report their beliefs in the current round. Thus, it appears that subjects' reported beliefs do reflect internalized beliefs since they are related from round to round without being listed on the computer screen.

With the use of these data now justified, I can combine the reported beliefs with the threshold distribution probabilities to directly calculate each i's subjective probability of

[18] See Chapter 13 in Greene (1997) for a discussion of autocorrelation and autocorrelation tests.

being pivotal in time t:

$$\Pr\left[piv_t|b_{it},T\right] = b_{it}(0)\Pr\left[t=1|T\right] + ... + b_{it}(4)\Pr\left[t=5|T\right].$$

Because the theoretical decision rule depends on a player's subjective beliefs about others' contributions, I can now use the reported beliefs to ascertain how closely the observed behavior reflects the game-theoretic decision rule that subjects contribute when $\Pr\left[piv_t|b_{it},T\right] \geq \frac{c}{v}$ and do not contribute otherwise.

Finding 3: *Letting reported beliefs proxy for true beliefs, subjects' actions are not consistent with expected payoff maximization.*

Table 4 details how frequently subjects' contributions matched this decision rule. 65 percent of all decisions are consistent with expected payoff maximization.[19] Only about 1 percent more are consistent in rounds 8 and higher. Note that deviations from the decision rule differ across sessions and (v,T)-combinations, with 55 percent to 80 percent consistent across sessions and 50 percent to 70 percent across (v,T)-combinations. When $v=6$ and the decision rule says "should not" contribute, then subjects are more likely to contribute, whereas when $v=3$ and the rule says "should not", then subjects are more likely to not contribute.

Finding 2 provides an initial answer to the second question posed above. The predictions are not strongly verified because subjects are not following the model's decision rule. But why are the predictions verified to any degree?

Finding 4: *Subjects are more likely to contribute when they believe they are more likely to be pivotal.*

Figure 3(a) plots three non-parametric fits of the probability of contribution for different values of $\left(\Pr\left[piv|b_{it},F\right] - \frac{c}{v}\right)$. I use the Epanechnikov kernel in the Nadaray-Watson kernel estimator under three different smoothing bandwidth parameters $h = 0.025$, 0.1, and 0.15

[19] This calculation uses all observations except the one with the incorrectly reported beliefs, thus leaving a total of 9629 observations.

(Härdle (1990)).[20] Denoting $x = \Pr\left[piv|b_{it}, F\right] - \frac{c}{v}$, where x ranges from -0.333 to 0.833 in the data, this estimator is

$$m_h\left(X_i, h\right) = \frac{\frac{1}{(h)(\#obs)} \sum_{obs} \frac{3}{4}\left(1 - \left(\frac{x_{obs}-X_i}{h}\right)^2\right) I\left(\frac{x-X_i}{h} \leq 1\right) a_{obs}}{\frac{1}{(h)(\#obs)} \sum_{obs} \frac{3}{4}\left(1 - \left(\frac{x_{obs}-X_i}{h}\right)^2\right) I\left(\frac{x-X_i}{h} \leq 1\right)}.$$

The curve labeled "EP Decision Rule" depicts the model's game theoretic decision rule. Figure 3(b) plots the $h = 0.1$ fit with 95 percent confidence intervals.[21]. The first thing to note is that subjects are more likely to contribute than not contribute even at many negative values of $\left(\Pr\left[piv|b_{it}, F\right] - \frac{c}{v}\right)$. This provides further evidence for rejecting the consistency of observed actions with the model's expected payoff maximization decision rule. Nonetheless, while expected payoff maximization is clearly rejected, note that Figure 3(a) also reveals that the likelihood of contributing increases in $\left(\Pr\left[piv|b_{it}, F\right] - \frac{c}{v}\right)$. The slope of the non-parametric fit is positive, with the estimated probability of contributing increasing from below 0.5 at $\left(\Pr\left[piv|b_{it}, F\right] - \frac{c}{v}\right) = -0.4$ to about 0.75 at high values of $\left(\Pr\left[piv|b_{it}, F\right] - \frac{c}{v}\right)$.

Overall, the subjects' actual decision rules and the model's decision rule have an important qualitative similarity and an important difference. The similarity is that subjects strategically consider their pivotalness when making contribution decisions. Because pivotalness is strategic in the sense that it depends on a subject's beliefs about others' actions (in all cases except $T = \{1, 2, 3, 4, 5\}$), subjects are playing strategically in a game theoretic sense. Moreover, they are responding to pivotalness even in the presence of threshold uncertainty. Thus, the model captures an important aspect of the subjects' strategic behavior. This finding is particularly striking since subjects were not directly asked to report

[20] A smaller bandwidth parameter implies that the only observations to receive weight are those closer to the point being estimated. While a smaller bandwidth implies greater precision in the sense of putting more weight on the more appropriate observations, if the bandwidth parameter is too small, then too few observations will given weight. By trial and error, I found $h = 0.025$ to be the smallest bandwidth that still includes a meaningful number of observations for most point estimates.

[21] To obtain better confidence intervals, I should compute bootstrap interval estimates. For statistical ease, however, I use the approximate confidence interval described by Härdle (1990, 100-101). The interval is $m_h(x) \pm \left(c_\alpha c_K^{1/2} \widehat{\sigma}(x)\right) / \sqrt{\left(nh\widehat{f}(x)\right)}$, where c_α is the $100 - \alpha$ quantile of the normal distribution, $c_K^{1/2}$ is a kernel constant, $\widehat{\sigma}(x)$ is the estimate of the standard deviation, and $\widehat{f}(x)$ is the estimate of the density. This confidence interval is hampered by a bias, but as we see from the graph, the bias must be large for consistency with EP maximization to be a legitimate possibility.

a probability of being pivotal. Had I asked them directly what the chances were that their own contribution was necessary to meet the threshold, then it is likely that this direct report of pivotalness would factor more heavily into their contribution decision, since directly asking them about pivotalness could unintentionally lead them to believe that pivotalness should determine factor into the contribution decision. The fact that the inferred subjective pivotalness does matter suggests that subjects consider their pivotalness of their own volition.

However, the main difference between actual behavior and the model is that subjects do not respond as sharply to pivotalness around the cutpoint $\frac{c}{v}$ as implied by the model. When near the cutpoint, an increase in pivotalness only marginally increases the (global) probability of contribution. This offers one explanation for why contributions under $T = \{1, 2, 3, 4, 5\}$ were lower than under $T = \{3\}$ in sessions 5, 6, and 7. When $T = \{1, 2, 3, 4, 5\}$, a contributor's probability of being pivotal is $\frac{1}{5}$ no matter what she thinks others will do. When $T = \{3\}$, the probability of being pivotal is the probability assigned to the event that exactly two others contribute. If this probability is greater than $\frac{1}{5}$, which will often be the case, and if players use a best response rule that is strictly monotonically increasing in $\left(\Pr\left[piv|b_{it}, F\right] - \frac{c}{v}\right)$ (as in the non-parametrically estimated function in Figure 4.1), then we will see more contributions under $T = \{3\}$ than $T = \{1, 2, 3, 4, 5\}$. More generally, if contributions depend not just on whether $\Pr\left[piv|b_{it}, F\right]$ is greater than $\frac{c}{v}$ but also on the difference between the two, then we will observe deviations from the model's predictions.

5 Conclusion

The theoretical results indicate that for highly valued public goods a widening of threshold uncertainty will increase individuals' probabilities of being pivotal, thereby driving up contributions. The experimental results moderately support these predictions. A widening of uncertainty often, but not always, results in movements in contributions in the expected manner. Although the subjects relate changes in threshold uncertainty into changes in pivotalness and consider pivotalness when making contribution decisions, they do not respond to pivotalness as sharply as the model implies. These last findings are obtained using data on subjects' subjective beliefs about other subjects' contributions.

The main implication of these findings is that whether or not threshold uncertainty hinders collective action will depend on the size of the benefits resulting from successful action and also on how individuals respond to pivotalness. Increases in threshold uncertainty may actually increase the likelihood of successful action when the benefits of successful collective action are large. However, because individuals do not respond to pivotalness to the degree implied by the model, this might only occur under small levels of threshold uncertainty. Threshold uncertainty will almost certainly hinder collective action when the value of the public good is low because wider uncertainty in this setting will lower individuals' probabilities of being pivotal. The risk of participating in a lost cause is then too high relative to the small potential gains.

It follows that groups facing threshold uncertainty will often need to undertake costly actions for collective action to succeed. One possibility would be the creation of mechanisms that exclude or punish non-participants. Another possibility, more in the spirit of this paper, would be the costly gathering of information that would reduce the variance in people's beliefs about the threshold, and this in turn raises a number of other strategic issues. For example, a group may actually prefer to not collect more information about the threshold if it is believed doing so will reduce the uncertainty so much that contributions will decrease. Also, a group leader with more precise information about the threshold may strategically reveal or not reveal her information in an attempt to obtain any surplus that can arise from contributions.

Future research should examine threshold uncertainty in these and other settings. Theoretical work should examine settings where individuals have private signals about the threshold, and an extension would allow some of those individuals to have noisier signals than others. Another setting would have a group leader who must choose whether or not to initiate costly information gathering. By examining these settings we can better understand how individuals' incentives to gather and share information differ across informational environments. Since much collective action occurs within formal groups or in the presence of other institutions, such work will lend insights into the actions taken by these groups to overcome the effects of threshold uncertainty. A different direction of research should focus more closely on individuals' contribution decisions. That individuals do not respond as sharply to pivotalness suggests the presence of other strategic or behavioral factors in the decision making process. Prior research suggests a number of possibilities, e.g., that individuals differ in risk attitudes, dynamic strategic play, and learning. Accounting for these will allow us to better explain the observed behavior.[22] These avenues of research will ultimately lead us to a more complete understanding of collective action.

6 References

Au, Winton. "Criticality and Environmental Uncertainty in Step-level Public Goods Dilemmas." *Group Dynamics: Theory, Research, and Practice*, 2004, (8), pp. 40-61.

Bagnoli, Mark and Barton Lipman. "Private Provision of Public Goods can be Efficient." *Public Choice*, 1992, (74), pp. 59-78.

Budescu, David; Rapoport, Amnon and Suleiman, Ramzi. 1995. "Common Pool Resource Dilemmas Under Uncertainty: Qualitative Tests of Equilibrium Solutions." *Games and Economic Behavior*, 1995, (10), pp. 171-201.

Camerer, Colin. "Individual Decision Making" in John Kagel and Alvin Roth, eds., *The Handbook of Experimental Economics.* Princeton, NJ: Princeton University Press, 1995, pp. 587-703.

Chwe, Michael Suk-Young. "Structure and Strategy in Collective Action." *American Journal of Sociology* , 1999, (105), pp. 128-156.

Chwe, Michael Suk-Young. "Communication and Coordination in Social Networks." *Review of Economic Studies*, 2000, (67), pp. 1-16.

[22] This direction of research appears very promising. In preliminary work along these lines, I find evidence of statistically significant heterogeneity across individuals. While the simple expected payoff maximization rule is consistent with only 65 percent of decisions (Table 4.4), accounting for individual fixed effects in probit regressions yield esimates that correctly predict over 80 percent of decisions. Moreover, using a grid procedure to estimate risk aversion and cooperation bias parameters yields estimates that correctly predict about 90 percent of decisions. Another line of research would look at the presence of dynamic strategies.

Dekel, Eddie and Piccione, Michele. "Sequential Voting Procedures in Symmetric Binary Elections." *Journal of Political Economy*, 2000, (108), pp. 34-55.

Gintis, Herbert. *Game Theory Evolving.* Princeton, NJ: Princeton University Press, 2000.

Granovetter, Mark. "Threshold Models of Collective Behavior." *American Journal of Sociology*, 1978, (83), pp. 1420-1443.

Greene, William. *Econometric Analysis.* Upper Saddle River, New Jersey: Prentice-Hall, Inc., 1997.

Gustafsson, Mathias; Biel, Anders and Gärling, Tommy. "Overharvesting of Resources of Unknown Size." *Acta Psychologica*, 1999, (103), pp. 47-64.

Härdle, Wolfgang. *Applied Nonparametric Regression.* Cambridge, UK: Cambridge University Press, 1990.

Lafont, Jean-Jaques. *The Economics of Uncertainty and Information.* Cambridge, MA: MIT Press, 1990.

Ledyard, John. "Public Goods A Survey of Experimental Research" in John Kagel and Alvin Roth, eds., *The Handbook of Experimental Economics.* Princeton, NJ: Princeton University Press, 1995, pp. 111-194.

Lichbach, Mark. "Contending Theories of Contentious Politics and the Structure-action Problem of Social Order." *American Review of Political Science*, 1998, (1), pp. 401-24.

McKelvey, Richard and Thomas Palfrey. "Quantal Response Equilibria for Normal Form Games." *Games and Economic Behavior*, 1995, (10), pp. 6-38.

Menezes, Flavio; Monteiro, Paulo and Temimi, Akram. "Private Provision of Discrete Public Goods with Incomplete Information." *Journal of Mathematical Economics*, 2001, (35), pp. 493-514.

Newbold, Paul. *Statistics for Business and Economics.* Englewood Cliffs, NJ: Prentice-Hall, Inc, 1995.

Nitzan, Shmuel and Romano, Richard. "Private Provision of a Discrete Public Good with Uncertain Cost." *Journal of Public Economics*, 1990, (42), pp. 357-370.

Nyarko, Yaw and Schotter, Andrew. "An Experimental Study of Belief Learning Using Elicited Beliefs." *Econometrica*, 2000, (70), pp. 971-1005.

Offerman, Theo. *Beliefs and Decision Rules in Public Good Games: Theory and Experiments.* Amsterdam: Tingergen Institute Research Series, No. 124, 1996.

Olson, Mancur. *The Logic of Collective Action.* Cambridge, MA: Harvard University Press, 1965.

Ostrom, Elinor. "Collective Action and the Evolution of Social Norms." *Journal of Economic Perspectives*, 2000, (14), pp. 137-158.

Palfrey, Thomas and Rosenthal, Howard. "Participation and the Provision of Discrete Public Goods: A Strategic Analysis." *Journal of Public Economics*, 1984, (24), pp. 171-193.

Palfrey, Thomas and Rosenthal, Howard. "Private Incentives in Social Dilemmas." *Journal of Public Economics*, 1988, (35), pp. 309-332.

Palfrey, Thomas and Rosenthal, Howard. "Testing Game-theoretic Models of Free-riding: New Evidence on Probability Bias and Learning," in Thomas Palfrey, ed., *Laboratory Research in Political Economy.* Ann Arbor, MI: University of Michigan Press, 1991, pp. 239-268.

Sandler, Todd. *Collective Action: Theory and Applications.* Ann Arbor, MI: University of Michigan Press, 1992.

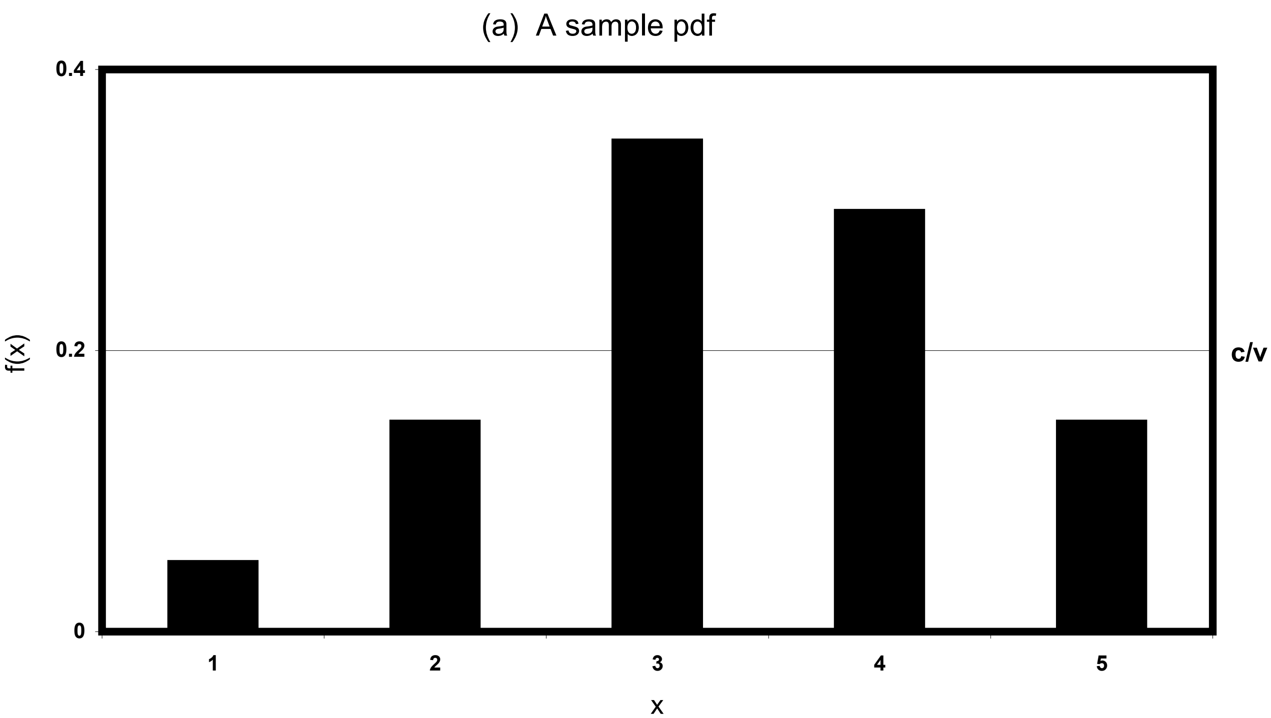
Suleiman, Ramzi. "Provision of Step-level Public Goods under Uncertainty: A Theoretical Analysis." *Rationality and Society*, 1997, (9), pp. 163-187.

Suleiman, Ramzi; Budescu, David and Rapoport, Amnon. "Provision of Step-level Public Goods with Uncertain Provision Threshold and Continuous Contribution." *Group Decision and Negotiation*, 2001, (10), 253-274.

Wit, Arjaan; Wilke, Henk. "Public Good Provision under Environmental and Social Uncertainty." *European Journal of Social Psychology*, 1998, (28), pp. 249-256.

Yin, Chien-Chung. "Equilibria of Collective Action in Different Distributions of Protest Thresholds." *Public Choice, 1998, (*97), pp. 535-567.

Figure 1: An Example for Finding Equilibria Graphically



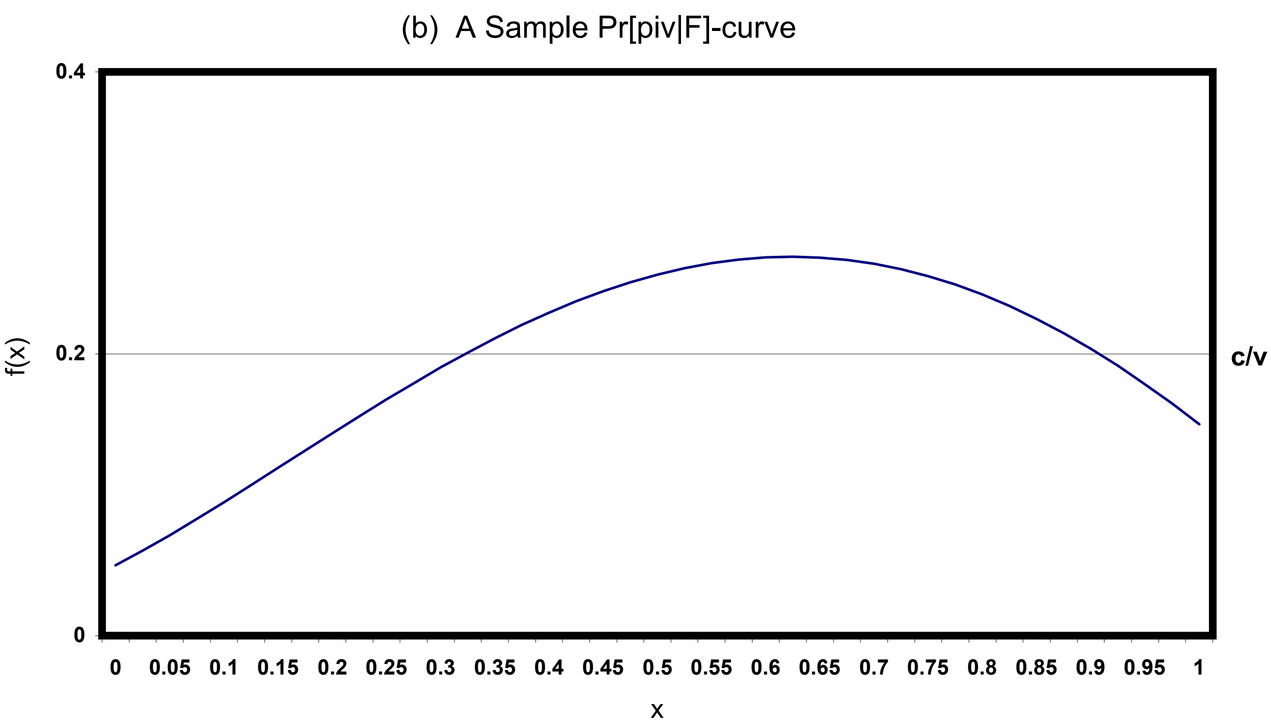
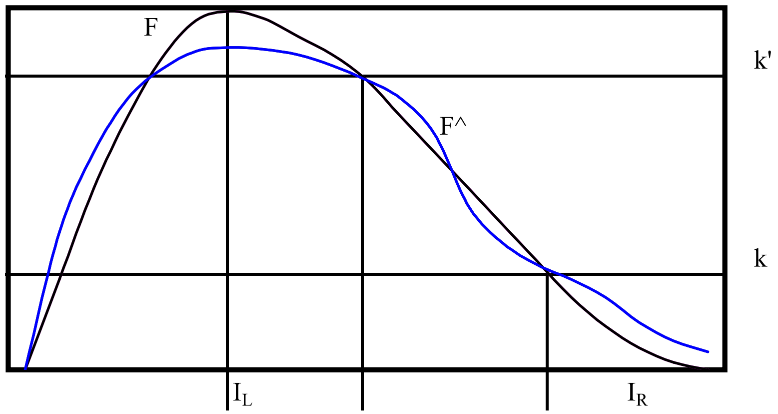
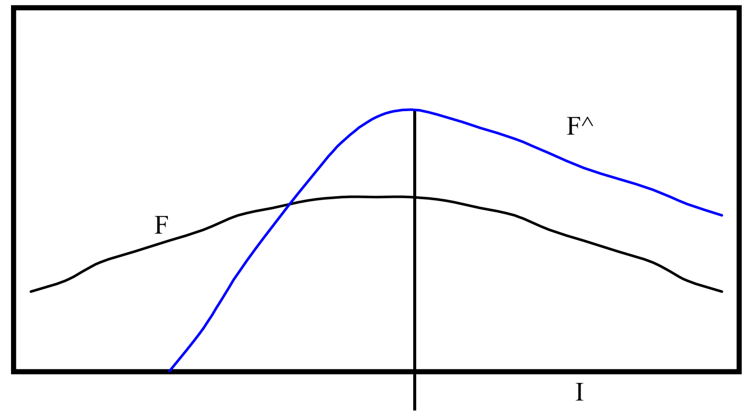


Figure 2: Illustrations of Interior Tails

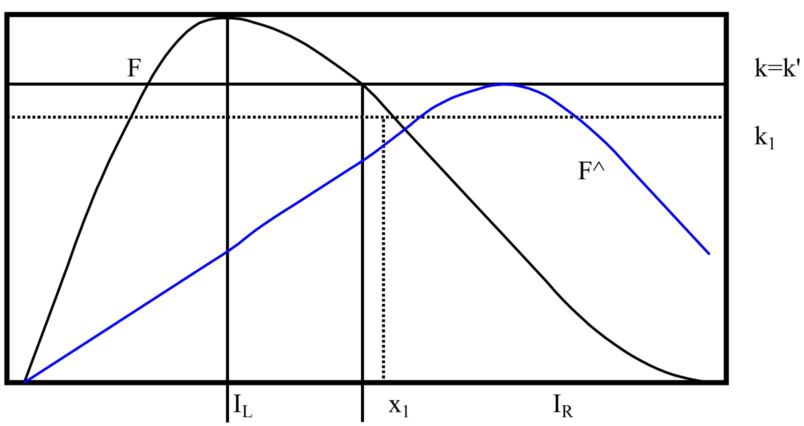
(a) Interior Tails Exist but do not Meet



(b) One PDF Above the Other



(c) Interior Tails Meet Under Single-crossing



(d) A Simple Mean Preserving Spread

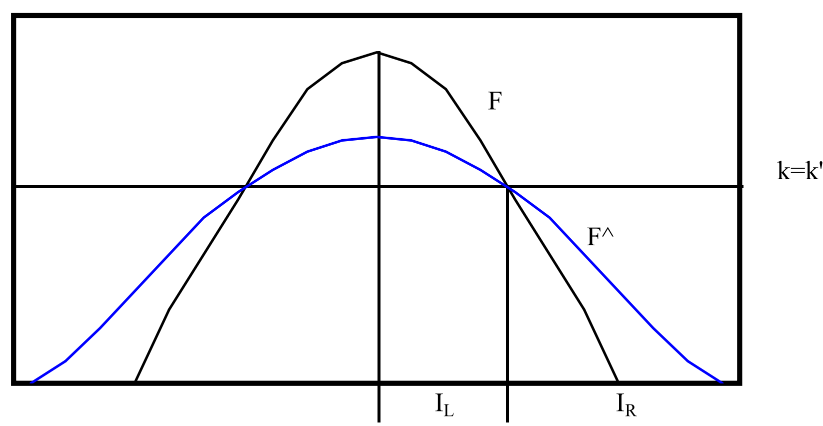


Table 1: Treatment and Session Descriptions

(a) Proportion of Contributions By Parameter Combinations

	v=3		v=6		
	T={3}	T={1,2,3,4,5}	T={3}	T={2,3,4}	T={1,2,3,4,5}
Mixed Equilibrium	0.62	0	0.79	0.84	1
Qualitative Prediction	decrease →		increase →		

(b) Treatments and Sessions

Treatment	Number of Sessions
(3,{3}) to (3,{1,2,3,45})*	1
(6,{3}) to (6,{2,3,4})*	1
(6,{2,3,4}) to (6,{3})*	1
(6,{3}) to (6,{1,2,3,4,5})*	3
(6,{1,2,3,4,5}) to (6,{3})*	1
(6,{3})	1
(6,{2,3,4})	3
(6,{1,2,3,4,5})	3

* When the threshold range switches, the first 15 rounds are under the first range, and the rest of the rounds are under the second range.

Table 2: Contribution Means

(a) Aggregated Contribution Means by (v, T)-combination

	v=3		v=6		
	T={3}	T={1,2,3,4,5}	T={3}	T={2,3,4}	T={1,2,3,4,5}
Mixed Equilibrium	0.62	0	0.79	0.84	1
Qualitative Prediction	decrease →		increase →		
All Sessions and Rounds	0.539	0.499	0.687	0.695	0.698
(Observations)	(375)	(375)	(3525)	(3225)	(4080)
All Sessions and Rounds 8+	0.533	0.460	0.687	0.690	0.712
(Observations)	(200)	(200)	(2090)	(2280)	(2680)

(b) Contribution Means by Session and (v, T)--combination for Rounds 8+

	v=3		v=6		
	T={3}	T={1,2,3,4,5}	T={3}	T={2,3,4}	T={1,2,3,4,5}
1. (3,{3}) to (3,{1,2,3,45})	0.535	0.460			
2. (6,{3}) to (6,{2,3,4})			0.605	0.655	
3. (6,{2,3,4}) to (6,{3})			0.675	0.721	
4. (6,{3}) to (6,{1,2,3,4,5})			0.733		0.750
5. "			0.708		0.688
6. "			0.638		0.575
7. (6,{1,2,3,4,5}) to (6,{3})			0.679		0.638
8. (6,{3})			0.710		
9. (6,{2,3,4})				0.686	
10. "				0.692	
11. "				0.692	
12. (6,{1,2,3,4,5})					0.791
13. "					0.710
14. "					0.711

Table 3: Measures of Reported Beliefs Movements

(a) Proportion of Time Reported Beliefs Moved in Direction Indicative of Beliefs Updating

	% of Time Reported Beliefs Average Moved towards Last Actual	% of Time Assigned Higher Percent to Last Actual
Overall	0.754	0.817
Rounds 8+	0.756	0.816
1. (3,{3}) to (3,{1,2,3,45})	0.764	0.791
2. (6,{3}) to (6,{2,3,4})	0.756	0.853
3. (6,{2,3,4}) to (6,{3})	0.793	0.852
4. (6,{3}) to (6,{1,2,3,4,5})	0.776	0.829
5.	0.730	0.806
6.	0.775	0.800
7. (6,{1,2,3,4,5}) to (6,{3})	0.785	0.852
8. (6,{3})	0.766	0.821
9. (6,{2,3,4})	0.748	0.822
10.	0.746	0.815
11.	0.742	0.810
12. (6,{1,2,3,4,5})	0.749	0.824
13.	0.727	0.781
14.	0.766	0.852

(b) Regressions of Mean Beliefs on Various Controls

	OLS	**1st AR(1)**	**2nd AR(1)**
intercept	0.4499	0.2624	0.2552
	(0.01782)	(0.0161)	(0.0284)
mean belief $_{t-1}$	0.8314	0.9011	0.8931
	(0.00649)	(0.0059)	(0.0104)
actual $_{t-1}$ - mean belief $_{t-1}$	0.1256	0.1246	0.1890
	(0.00350)	(0.0033)	(0.0062)
Parameter Round	--	--	-0.0001
			(0.0022)
(mean belief $_{t-1}$)*(Parameter Round)	--	--	0.0009
			(0.0008)
(actual $_{t-1}$ - mean belief $_{t-1}$)*(Parameter Round)	--	--	-0.0055
			(0.0004)
R^2	0.612	0.625	0.634
Durbin-Watson	2.276	--	--

Table 4: Proportion Contributions Consistent with Expected Payoff Maximization

	Should & Did (1)	Should & Did not (2)	Indifferent (3)	Should not & Did (4)	Should not & Did not (5)	Consistent (1)+(3)+(5)	Consistent Rounds 8+
(a) Overall	0.574	0.249	0.014	0.099	0.065	0.652	0.669
(b) By (v,T)-combination							
(3,{3})	0.360	0.227	0.000	0.179	0.235	0.595	0.620
(3,{1,2,3,4,5})	--	--	--	0.499	0.501	0.501	0.540
(6,{3})	0.463	0.193	0.000	0.224	0.121	0.584	0.598
(6,{2,3,4})	0.629	0.272	0.049	0.032	0.016	0.695	0.698
(6,{1,2,3,4,5})	0.698	0.302	--	--	--	0.698	0.702
(c) By Session							
1. (3,{3}) to (3,{1,2,3,45})	0.180	0.113	0.000	0.339	0.368	0.548	0.580
2. (6,{3}) to (6,{2,3,4})	0.551	0.319	0.015	0.077	0.039	0.604	0.618
3. (6,{2,3,4}) to (6,{3})	0.583	0.209	0.022	0.106	0.080	0.686	0.690
4. (6,{3}) to (6,{1,2,3,4,5})	0.558	0.236	0.000	0.160	0.046	0.604	0.625
5.	0.553	0.230	0.000	0.152	0.064	0.618	0.627
6.	0.511	0.354	0.000	0.080	0.054	0.566	0.590
7. (6,{1,2,3,4,5}) to (6,{3})	0.563	0.279	0.000	0.089	0.069	0.632	0.648
8. (6,{3})	0.414	0.151	0.000	0.297	0.138	0.552	0.567
9. (6,{2,3,4})	0.629	0.264	0.066	0.027	0.014	0.709	0.700
10.	0.640	0.257	0.047	0.024	0.032	0.719	0.718
11.	0.636	0.284	0.044	0.021	0.015	0.695	0.696
12. (6,{1,2,3,4,5})	0.793	0.207	--	--	--	0.793	0.791
13.	0.708	0.292	--	--	--	0.708	0.710
14.	0.705	0.295	--	--	--	0.705	0.711

Figure 3: Non-parametric Regressions