Managing Exotic Pests under Uncertainty:
Optimal Control Actions and Bioeconomic Investigations[1]

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Abstract

Using a real options framework in continuous time, we examine two problems associated with the management of exotic pests under uncertainty. First, we analyze the optimal timing of investing in a control action. Given fixed control costs, we obtain a closed-form solution for when to control an exotic pest whose density varies randomly. Second, we define an analytical framework for finding the optimal expenditures for obtaining the relevant bioeconomic data. Our numerical results, based on a wide range of bioeconomic parameters, highlight the importance of accounting for uncertainty and irreversibility when developing simple rules to manage exotic pests.

Key words: exotic species; bioeconomic investigations; uncertainty; irreversibility; real options.

JEL classification: D8, Q2

1. Introduction

The number of invasive species, pests introduced from elsewhere, is increasing worldwide, and contributes to biodiversity loss and environmental change (e.g., Wilcove et al. 1998, Sala et al. 2000, Lodge 2001, Leung et al., 2002). At least 4500 species, including plants and animals, for instance, are estimated to be non-indigenous in the United States (Rosenzweig, 2001). In response to this perceived risk, President Clinton signed in February 1999 Executive Order 13112 to prevent "the introduction of invasive species and provide for their control and to minimize the economic, ecological, and human health impacts that invasive species cause." And while EO 13112 stresses that exotic invaders are costly to ignore, it also emphasizes the stringency of budget constraints since control actions are "*subject to the availability of appropriations, and within Administration budgetary limits*" [emphasis added]. Since not all invaders become pests, it is important to invest wisely in the collection of relevant bioeconomic parameters (e.g., data on pest damage costs and pest biology) to know when to control, if ever. The complexity of this problem is compounded by the interplay between uncertainty and irreversibility. Uncertainty arises in the evolution of the pest population and in the duration of the bioeconomic investigations; irreversibility arises in pest damages (at least partial) and in bioeconomic or control expenditures.

In this paper, we address the issues of uncertainty and irreversibility, both of which have received little attention in the literature on the economics of exotic species. To account for uncertainty and irreversibility, we propose a real options framework in continuous time.[1] Our first model explores the decision of when, if ever, to control an exotic species invading a given territory, assuming that the relevant bioeconomic parameters are known. This approach allows

1

us to find a closed-form action rule, i.e., a threshold set by biological and economic parameters, that triggers the control of an invasive species. This threshold is similar to the exercise of an option. As noted by the National Invasive Species Management Plan (2001), understanding the timing of control measures in an uncertain environment requires an analytical framework for invasive species so control strategies can be prioritized and targeted appropriately. The theory of real options provides resource managers with a coherent framework to balance urgency of action and neglect. We examine how our threshold varies with different bioeconomic parameters by conducting a sensitivity analysis. We find the control threshold varies negligibly with the efficacy of a control measure.

Our second model, which encompasses the first one, examines the optimal effort to dedicate to gathering bioeconomic data on damage costs and pest biology. B*ioeconomic investigations* are typically necessary to gather the necessary biological and economic data for a new, exotic invader. We assume the time necessary to obtain reasonable bioeconomic parameters is a random variable whose mean can be decreased by increasing the level of expenditures for the bioeconomic investigations. Our numerical results illustrate why accounting for uncertainty and irreversibility matters for developing simple rules to manage exotic species.

This paper is organized as follows. In Section II, we present a general framework to find the control threshold and the optimum amount to spend on biological investigations when the pest density follows a general diffusion process. In Section III, we derive results for the special case in which the pest density follows a geometric Brownian motion (GBM). Section IV illustrates our models numerically for a wide range of bioeconomic parameters. Section V concludes and offers our suggestions for future work.

2. Defining a framework

We begin by presenting a general real options framework to better understand the nature of two important facets of exotic species management—the optimal investment to control the spread of an invader, and the optimum investment in biological investigations to learn more about the exotic pest problem. Consider each in turn.

2.1 The invasive species control problem

To help motivate our approach, consider the parallel between controlling an exotic pest and investing in a financial asset. First, each decision is (at least partly) irreversible. Damage caused by the pest cannot be undone, and a bad investment cannot usually be recovered. Second, decisions are made under uncertainty. Competitors, technological breakthroughs, changes in demand, or natural catastrophes affect randomly the value of an investment. Similarly, the future levels of the pest density, and future damages, are stochastic because of fluctuations in predator activity, the weather, and the availability of food. Finally, both investing and applying pest control can be delayed under unfavorable conditions. These three characteristics call for the use of the theory of real options (see Dixit and Pyndick, 1994).

If we view the particular ecosystem at risk from an exotic pest as *an asset*, in which one can invest by controlling the pest population, the resource manager is holding a *compound option*. This compound option gives the right but not the obligation to apply a control measure, and it never expires if the resource manager's time horizon is infinite, which we assume. Moreover, when this option is exercised, it gives rise to a similar option at the new value of the pest density.[2]

Just as in the Faustmann problem in forestry (Clark 1976, Ch. 8), the resource manager

faces the same type of problem after the application of each control measure. We invoke Bellman's optimality principle to infer that, if a unique solution exists, the optimal policy is to apply a control measure as soon as the exotic pest density X reaches size x^*. We choose x^* to minimize the present value of expected damages plus control costs. This threshold x^* separates the "low" and "high" values of X. For "low" values of X, waiting is optimum; for high values, acting immediately is best. The resource manager therefore faces a *compound stopping problem*.

Now consider the modeling formalities of the control problem in a general real options setting. Our objective is to define the appropriate objective function and conditions for optimum control timing. Consider an exotic pest population whose average density over a given area, X, varies randomly. Assume X follows the diffusion process[3]

$$dX = \mu(X)dt + \sigma(X)dz, \tag{1}$$

which is defined over the interval $[0,+\infty)$. The infinitesimal mean $\mu(x)$ and variance $\sigma(x)>0$ of X are continuous functions of x, and dz is an increment of a standard Wiener process (Karlin and Taylor 1981). When $X=x$, the flow of damages per unit time caused by the pest is denoted by $F(x)$, an increasing function of x.[4]

The manager may invest in control actions, such as introducing a predator, releasing sterile pests, applying a pesticide, or spraying an herbicide, at a cost $C_c(x)$ per action, to reduce the density of the exotic pest and the damages it causes. We make two simplifying assumptions. First, to have a single control variable, we suppose the level of our most effective control measure is fixed. This is the case for pesticides, for example, since dosage is frequently prescribed by law. Second, we suppose that the control measure acts immediately, which is satisfactory for most pesticides or herbicides, fast acting predators, or the release of sterile

individuals for pests with short life spans. In addition, as is typical in practice, our control measure only reduces the exotic pest population to a fraction ω of its value before application, so control needs to be repeated over time. The parameter ω represents the efficacy of control. Complete control inefficacy ($\omega=1$) and complete eradication ($\omega=0$) are special cases. Alternatively, the exotic pest may become extinct on its own after an initial population outburst if it fails to adapt to its new environment.

We denote by $V_c(x|y)$ the value function for this problem when the current pest population is y and control takes place at x. From our discussion, $V_c(x|y)$ has three components: (1) the present value of the expected flow of pest damages between the moment when the starting pest density is y and the moment at which it first reaches either x or 0 (and becomes extinct); (2) the discounted pest control costs at x; and (3) the discounted value function immediately after the application of a control measure. The resource manager's problem is to minimize $V_c(x|y)$ by choosing x, the level of the pest population at which controls should be applied, given that the current pest density is $y<x$, where

$$V_c(x \mid y) = \Gamma^A(x \mid y) + D_{0,x|y}^A \left\{ C_c(x) + V_c(x \mid x\omega) \right\}. \tag{2}$$

In equation (2):

- $\Gamma^A(x \mid y) = E_X \left\{ \int_0^{T_{0,x|y}} F(X)e^{-\rho t} dt \right\}$ is the expected value of discounted pest damages, E_X is the expectation operator with respect to X, $T_{0,x|y}$ is the random duration between the moment where X equals y and the first time X hits either 0 or x; and ρ is the relevant discount rate, appropriately adjusted for risk; 0 does not appear explicitly in $\Gamma^A(x \mid y)$ to lighten our

notation;

- $D_{0,x|y}^A = E\left(e^{-\rho T_{0,x|y}^A}\right)$ is the expected discount factor; $T_{0,x|y}^A$ is the first time X hits x starting

  from y *conditional on X hitting x before 0* since 0 is absorbing; and

- $C_c(x)$ is the cost of controlling the exotic pest when $X=x$. Applying a control measure brings

  the pest density to a fraction ω of its value before application.

We show in the appendix how to derive $\Gamma^A(x \mid y)$ and $D_{0,x|y}^A$. Similarly, the value function

immediately after controlling the pest is

$$V_c(x \mid x\omega) = \Gamma^A(x \mid x\omega) + D_{0,x|x\omega}^A \left\{C_c(x) + V_c(x \mid x\omega)\right\}, \tag{3}$$

From (3), we can derive an expression for $V_c(x \mid x\omega)$. When we plug this expression of

$V_c(x \mid x\omega)$ into (2) and rearrange terms, the value function becomes

$$V_c(x \mid y) = \Gamma^A(x \mid y) + \frac{D_{0,x|y}^A}{1 - D_{0,x|x\omega}^A}\left[C_c(x) + \Gamma^A(x \mid x\omega)\right]. \tag{4}$$

The first term on the right side of (4) is the present value of expected pest damages until the next

control given the current pest density is y. The other terms on the right side of (4) represent the

present value of the sum of all future control costs and expected pest damages.

As in Saphores (2003), if we derive a "standard" first order necessary condition for this

problem, the threshold for applying the control depends on y, the current value of the pest

density, which changes randomly. This implies we have a moving target problem. To find the

actual first order necessary condition, we write it at the optimum, i.e., we impose that $y = x = x^*$,

and express it in terms of elasticities (as in Dixit et al., 1999). This approach leads to:

$$\frac{x\dfrac{\partial \Gamma^A(x\mid y)}{\partial x}}{\dfrac{C_c(x)+\Gamma^A(x\mid x\omega)}{1-D_{0,x\mid x\omega}^A}} = \varepsilon^{\Sigma D^A}+\varepsilon^{C\Gamma^A}, \text{ for } y = x = x^*. \tag{5}$$

On the right side of equation (10), $\varepsilon^{\Sigma D^A} \equiv \dfrac{\partial \ln\left(\dfrac{D_{0,x\mid y}^A}{1-D_{0,x\mid x\omega}^A}\right)}{\partial \ln(x)}\Big|_{y=x=x^*}$ and

$\varepsilon^{C\Gamma^A} \equiv \dfrac{\partial \ln\left(C_c(x)+\Gamma^A(x\mid x\omega)\right)}{\partial \ln(x)}\Big|_{x=x^*}$ are the elasticity of the sum of discount factors

incorporating the possibility of pest extinction between controls, and the elasticity of control

costs plus the flow of pest damages between two successive applications of the control measure.

The left side of equation (5) is the average damage contribution between two consecutive

controls (numerator) divided by the present value of the sum of all future damage and control

costs (denominator). It scales the impact on damages of a marginal change in the timing of the

next control with the present value of all future damage and control costs. Overall, equation (5)

balances three effects of a change in the timing of control: (1) the flow of damages until the next

control; (2) delaying all future controls; and (3) controlling costs and damages between controls.

Let us consider expression (5) in more detail as the two measures of elasticity provide

insight into the timing threshold. First consider the elasticity of the discount factor. Waiting to

apply the control delays incurring the control costs but increases the flow of pest damages; how

long to wait depends on the elasticity of the discount factor to the control threshold. If a small

increase in the threshold (i.e., waiting longer) causes the discount factor to decrease a lot, future

control costs are discounted more heavily (they occur at discrete times in the future) whereas the

present value of the flow of pest damages does not change much. In that case, the manager is

more likely to wait to control. Conversely, if a small increase in the threshold causes the discount factor to decrease very little, future control costs are almost unchanged whereas the present value of the flow of pest damages increases, so the manager is more likely to control now.

Now consider the second elasticity—control costs plus the between-control flow of pest damages. The manager's decision on how long to wait depends on the elasticity of the control-damages to the control threshold. If control and damage costs are very responsive to waiting (i.e., they increase a lot, possibly because the pest growth is explosive), the manager controls now. Conversely, if the sum of control and damages costs changes little with a small increase in x^*, she waits to control. Formulating the timing problem in terms of elasticities provides a relatively intuitive and consistent rule of thumb for managers that helps formulate their thinking about timing based on empirical or judgmental estimates of the relative magnitudes of elasticity measures.

If there is no risk of natural extinction and the pest population "rebounds" as soon as it reaches a lower value $\varepsilon > 0$ (due, for example to a steady pest influx), a similar analysis leads to

$$\frac{x \dfrac{\partial \Gamma^R (x \mid y)}{\partial x}}{\dfrac{C_c(x) + \Gamma^R(x \mid x\omega)}{1 - D^R_{\varepsilon, x\mid x\omega}}} = \varepsilon^{\Sigma D^R} + \varepsilon^{C\Gamma^R}, \text{ for } y = x = x^*, \tag{6}$$

where now:

- $\Gamma^R(x \mid y) = E_X \left\{ \displaystyle\int_0^{T^R_{\varepsilon, x\mid y}} F(X) e^{-\rho t} dt \right\}$ is the expected value of discounted pest damages, and

  $T^R_{\varepsilon, x\mid y}$ is the random duration between the moment where X equals y and hits x for the first

time, given that X is reflected at ε; again, ε does not appear explicitly in $\Gamma^R(x \mid y)$ to lighten our notation;

- $D_{\varepsilon,x|y}^R = E\left(e^{-\rho T_{\varepsilon,x|y}^R}\right)$ is the expected discount factor;

- $\varepsilon^{\Sigma D^R} \equiv \dfrac{\partial \ln\left(\dfrac{D_{\varepsilon,x|y}^R}{1 - D_{\varepsilon,x|x\omega}^R}\right)}{\partial \ln(x)}\Big|_{y=x=x^*}$ is the elasticity of the sum of discount factors given no possibility of pest extinction between controls; and

- $\varepsilon^{C\Gamma^R} \equiv \dfrac{\partial \ln\left(C_c(x) + \Gamma^R(x \mid x\omega)\right)}{\partial \ln(x)}\Big|_{x=x^*}$ is the elasticity of control costs plus the flow of pest damages between two successive applications of the control measure.

The interpretation of Equation (6) is similar to that of Equation (5). The special case when the pest can be eradicated (ω=0) can readily be derived from the above analysis.

2.2 The bioeconomic investigation problem

We have so far assumed the exotic pest's bioeconomic parameters are known (i.e., biological parameters describing the evolution of the pest population and economic parameters for the damage function and the control costs) to derive x^*, the threshold above which a control measure should be taken. In practice, however, this is unlikely to be the case so resources need to be devoted to evaluating the relevant bioeconomic parameters for newly discovered exotic species. As a result, a time lag exists between the first detection of an exotic species and the moment when a control measure can be considered, but not necessarily applied if the pest density is below the optimal action threshold x^*. The importance of time lags in stochastic investment

problems has been better understood since the work of Bar-Ilan and Strange (1996); it seems particularly relevant to explicitly model a time lag in this context because allowing for a long time period between the detection of an exotic pest and the possibility of its control may entail large damages.

To formalize this intuition, we denote by T the time to find satisfactory bioeconomic parameters. Suppose T is a continuous random variable whose mean can be decreased by increasing the level of expenditures on bioeconomic investigations. Moreover, denote by x_d the density at which the exotic pest is first detected. The new value function for this extended problem, $V_b(B)$, depends only on how much the resource manager decides to dedicate to bioeconomic investigations.[5] Our new value function $V_b(B)$ has three components: 1) the present value of the cost of bioeconomic investigations (B); 2) the present value of expected damages occurring during these investigations; and 3) the present value of successive control costs and of expected pest damages in-between:

$$V_b(B) = B + \int_0^{+\infty}\left[E_{X|x_d}\left\{ \int_0^t F(X)e^{-\rho\tau}d\tau \right\} + e^{-\rho t}\left(\int_0^{x^*} V_c(x^* \mid \xi)p(\xi,t;x_d)d\xi \right.\right.$$

$$\left.\left. + \sum_{j=1}^{+\infty} \int_{x^*\omega^{1-j}}^{x^*\omega^{-j}} \left[jC_c(\xi) + V_c(x^* \mid \xi\omega^j) \right] p(\xi,t;x_d)d\xi \right) \right] g(t;B)dt. \tag{7}$$

where:

- $E_{X|x_d}$ is the expected value with respect to X given that $X(0)=x_d$;

- $p(\xi,t;x_d)$ is the density function of X at point ξ and at time t if $X(0)=x_d$;

- $g(t;B)$ is the value at t of the density function of T. We suppose that the mean of T is a strictly decreasing function of B and that its variance is a non-increasing function of B;

and

- The term $\sum_{j=1}^{+\infty} \int_{x^*\omega^{1-j}}^{x^*\omega^{-j}} \left[jC_c(\xi) + V_c(x^* \mid \xi\omega^j) \right] p(\xi,t;x_d)d\xi$ corresponds to the case in which

    the pest density exceeds x^* once a control measure can be taken so the control measure

    has to be applied repeatedly to bring the pest population between 0 and x^*. j indicates the

    number of times the manager has to apply the control to bring the pest density between 0

    and x^* given that its value is greater than x^* at the end of the bioeconomic investigations.

The resource manager's problem is to minimize $V_b(B)$ by choosing B. The first order necessary

condition for (7) (not shown) states that two marginal changes need to sum to zero for the

duration of bioeconomic investigations to be optimal: (1) the marginal expenditure on these

investigations (equaling unity here); and (2) the marginal change in the sum of the present values

of expected damages during and after these investigations plus the successive control costs.

3. A specific case

To gain further insight into the properties of our real options models, let us now consider a

specific diffusion process. Suppose X follows the geometric Brownian motion (GBM)[6]

$$dX = \mu X dt + \sigma X dz, \tag{8}$$

where $\mu > 0$ and $\sigma > 0$ are the infinitesimal trend and volatility parameters (see Dixit and

Pindyck 1994). Since diffusions are Markovian (Karlin and Taylor 1981), the future distribution

of X depends only on its present value (and not on its past values).

This specific formulation implies that the pest density tends to increase ($\mu > 0$) around an

exponential trend. It is reasonable for invasions by pests who have multiple generations each

year and pose a risk of outbreak (such as mites, aphids, or trips). Examples of applications of the

GBM model to pest problems include Saphores (2000) and Sunding and Zivin (2000).

Assuming X follows a GBM also allows the derivation of a closed-form expression for the threshold x^*. This simplifies the sensitivity analysis for key bioeconomic parameters of the optimum timing of control and of the optimal level of bioeconomic expenditures.

To facilitate the presentation of our results, it is convenient to introduce two dimensionless parameters such that the results are independent of the units eventually selected

$$\eta \equiv 1 - \frac{2\mu}{\sigma^2}, \quad \lambda = \frac{\rho}{\mu}. \tag{9}$$

As σ increases from 0^+ to $+\infty$ (holding μ constant), η varies from $-\infty$ to 1.

In general, we expect damages to increase with the density of the pest population, so we suppose that the flow of pest damages is represented by the tractable form

$$F(x) = Fx^\delta, \tag{10}$$

where $F > 0$ is a scaling coefficient and $\delta \geq 1$. $F(x)$ is therefore a strictly convex function.

Moreover, we assume the pest control cost function is constant:

$$C_c(x) = C. \tag{11}$$

C represents the cost of treating the area under the control of the resource manager. This restriction is reasonable for control measures when dosage is fixed, e.g., for pesticides or herbicides.

To further simplify our problem, we suppose that there is no minimum pest population size so natural extinction is impossible because 0 is unattainable in finite time for the GBM (see Karlin and Taylor 1991).

In the following, to simplify our notation, we drop the subscript "R" and omit to

explicitly include 0 or ε. We thus write $D_{x|y}$ instead of $D_{0,x|y}^R$, $\Gamma(x\,|\,y)$ instead of $\Gamma^R(x\,|\,y)\dots$

3.1 The optimum control threshold

Given $0 < y \le x$, when we solve (A.4) and (A.5) for the expected discount factor, we obtain

$$D_{x|y} = \left(\frac{y}{x}\right)^{\theta},$$
(12)

where the bioeconomic meta-parameter θ is

$$\theta = \frac{-(\mu - 0.5\sigma^2) + \sqrt{(\mu - 0.5\sigma^2)^2 + 2\rho\sigma^2}}{\sigma^2} = \frac{\eta}{2} + \sqrt{\frac{\eta^2}{4} + \lambda(1-\eta)}.$$
(13)

$\theta > 0$ since $\eta \in (-\infty, 1)$ and $\lambda > 0$ (since $\mu > 0$).

In addition, from (A.1)-(A.2) we get an explicit expression for $\Gamma(x\,|\,y)$:[7]

$$\Gamma(x\,|\,y) = \frac{Fx^{\delta}}{\rho\kappa}\left[\left(\frac{y}{x}\right)^{\delta} - \left(\frac{y}{x}\right)^{\theta}\right],$$
(14)

where θ is given by (13) and

$$\kappa = 1 - \frac{\delta}{\rho}\left[\mu + \frac{\sigma^2}{2}(\delta - 1)\right] = 1 - \frac{\delta}{\lambda}\left[1 + \frac{\delta - 1}{1 - \eta}\right].$$
(15)

$\Gamma(x|y)$ is well defined for all levels of the pest volatility parameter σ because damages are computed over a finite (albeit random) time. To see this, observe from (15) that $\theta = \delta$ when $\kappa = 0$,

i.e., when $\sigma = \sqrt{2\dfrac{\rho - \mu\delta}{\delta(\delta - 1)}}$. For larger values of σ, $\kappa < 0$ and $\left(\dfrac{y}{x}\right)^{\delta} - \left(\dfrac{y}{x}\right)^{\theta} < 0$, so $0 < \Gamma(x|y) < +\infty$.

In (14), $\rho\kappa = \rho - \delta\{\mu + 0.5\sigma^2(\delta - 1)\}$ is an interest rate adjusted two ways for pest damages: first, through the rate of growth of the pest population (μ); and second, through the pest volatility

(σ). Increasing σ decreases κ since the pest damage function is assumed strictly convex ($\delta>1$).

When we introduce (12) and (14) in (4), we get

$$V_c(x \mid y) = \frac{Fx^\delta}{\rho\kappa}\left[\left(\frac{y}{x}\right)^\delta - \left(\frac{y}{x}\right)^\theta\right] + \frac{\left(\frac{y}{x}\right)^\theta}{1-\omega^\theta}\left[C + \frac{Fx^\delta}{\rho\kappa}(\omega^\delta - \omega^\theta)\right],$$ (16)

so the first order necessary condition at $x=y=x^*$, which is given here by (6), leads to the unique possibility for x^*:[8]

$$x^* = \left[\frac{\rho C}{F}\frac{\theta\kappa}{(\theta-\delta)(1-\omega^\delta)}\right]^{\frac{1}{\delta}}.$$ (17)

Expression (22) represents our optimal control threshold as a function of five dimensionless parameters: $\frac{\rho C}{F}$, η, λ, δ, and ω.[9]

A comparative static analysis shows:

- As expected, x^* increases with the control cost C and decreases with the scaling factor of damage costs, F. Only the ratio $\frac{\rho C}{F}$ matters, however, in the determination of x^*;

- x^* is an increasing function of ω—as the efficacy of the control measures diminishes (as ω is higher), it is optimal to wait longer to control. A less efficient control measure is similar to a more costly one; and

- x^* is a decreasing function of μ, the infinitesimal trend of the pest process. When the pest grows faster, the present value of future damages increases so it is best to act sooner.

   Indeed, $\dfrac{dx^*}{d\mu} = -\dfrac{\theta-\delta}{\kappa}\dfrac{\theta\sigma^2 x^*}{\rho(\sigma^2\theta^2 + 2\rho)} < 0$ since $\theta - \delta$ and κ have the same sign.[10] This

result does not depend on the assumption that $\delta > 1$.

The impact of the other parameters, the discount rate (ρ), the damage function exponent (δ), and the volatility coefficient of the pest population (σ), is less tractable, so we resort to numerical investigations.

3.2 The optimum bioeconomic expenditures

Once we know the optimal threshold x^*, we solve for the optimum expenditures on bioeconomic investigations to determine the exotic pest's bioeconomic parameters: the pest density trend μ and volatility σ, the characteristics of the damage function F and δ, as well as the cost of control C and its efficacy ω.

We first derive an expression for $E_{X|x_d} \int_0^t FX^\delta e^{-\rho\tau} d\tau$, which is the present value of expected pest damages during the bioeconomic investigations. Since X follows a GBM with trend μ and volatility σ, we know from Dixit and Pindyck (1984) that X^δ follows a GBM with drift $\delta[\mu + 0.5\sigma^2(\delta-1)]$ and volatility $\delta\sigma$. Therefore, $E_{X|x_d} X^\delta = x_d^\delta e^{\delta[\mu+0.5\sigma^2(\delta-1)]\tau}$, and

$$E_{X|x_d} \int_0^t FX^\delta e^{-\rho\tau} d\tau = Fx_d^\delta \frac{1-e^{-\rho\kappa t}}{\rho\kappa}, \tag{18}$$

where κ is defined by (15). In addition, the density function of $X(t)$ at ξ if $X(0)=x_d$ is

$$p(\xi,t;x_d) = \frac{1}{\xi\sigma\sqrt{t}} \varphi\left(\frac{Ln(\xi)-Ln(x_d)-\mu t}{\sigma\sqrt{t}}\right), \tag{19}$$

where $\varphi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$ is the standard normal density.

Finally, we suppose $\ln(\rho T)$ is normally distributed with mean $a + bB^{-1}$ and standard deviation s, where a, $b>0$, and $s>0$ are known constants.[11] Then the expected value of T when B is very large and s is very small is $E(T) = \rho^{-1}\exp(a)$. The parameter b characterizes the resource manager's ability to shorten bioeconomic investigations by investing more, and s describes the uncertainty surrounding these investigations. As a result, we have

$$g(t;B) = \frac{1}{ts}\varphi\left(\frac{\ln(\rho t) - (a + bB^{-1})}{s}\right).$$ (20)

When we insert expressions (18) to (20) into (7) and reorganize terms, we obtain the normalized value function[12]

$$\frac{V_b(B)}{C} = \frac{B}{C} + \int_0^{+\infty}\frac{\Lambda(u)}{us}\varphi\left(\frac{\ln(u) - (a + bB^{-1})}{s}\right)du,$$ (21)

where $u = \rho t$ is a dimensionless integration variable. Here $\Lambda(u)$ is given by[13]

$$\Lambda(u) = \frac{Fx_d^{\delta}}{\rho C}\frac{1 - e^{-\kappa u}}{\kappa} + e^{-u}\left(\frac{F}{\rho C}\frac{x^{*\delta}}{\kappa}J_0^{\delta}(u) + \left[\frac{F}{\rho C}\frac{x^{*\delta}}{\kappa}(\omega^{\delta} - 1) + 1\right]\frac{J_0^{\theta}(u)}{1 - \omega^{\theta}}\right.$$
$$\left. + \sum_{j=1}^{N}\left\{jI_j^0(u) + \frac{F}{\rho C}\frac{x^{*\delta}}{\kappa}I_j^{\delta}(u) + \left[\frac{F}{\rho C}\frac{x^{*\delta}}{\kappa}(\omega^{\delta} - 1) + 1\right]\frac{I_j^{\theta}(u)}{1 - \omega^{\theta}}\right\}\right),$$ (22)

$\Lambda(u)$ is the present value, divided by C, of the sum of expected damages during (1st term) and after the bioeconomic inquiry, including control costs (terms in factor of e^{-u}). N is a truncation parameter for the infinite sum in (7) because its terms quickly vanish to 0 for the range of parameters explored in this paper. In (22), for any real $a>0$ and any integer j, we have

$$J_j^a(u) = \left(\frac{x_d}{x^*}\right)^a e^{\left(1 + \frac{a}{1-\eta}\right)\frac{a}{\lambda}u}\Phi\left(\sqrt{\frac{\lambda(1-\eta)}{2u}}\left[\ln\left(\frac{x^*}{x_d\omega^j}\right) - \left(1 + \frac{2a}{1-\eta}\right)\frac{u}{\lambda}\right]\right),$$ (23)

16

where $\Phi(z)$ is the value at z of the standard normal cumulative distribution function, and the efficacy of efforts between the actions

$$I_j^a(u) = \omega^{aj}\left(J_j^a(u) - J_{j-1}^a(u)\right). \tag{24}$$

Our value function $V_b(B)$ defined by expression (26) needs to be solved numerically for the optimal value of B.

4. A numerical illustration

We now consider a numerical example to illustrate the properties of our framework for both the control and bioeconomic investigation problems. First, we describe the data, and then our numerical results.

4.1 Data

For our baseline, we choose an infinitesimal trend of μ=0.08 per year, a damage exponent of δ=2, a control efficacy of ω=0.01 (so 99% of the pest is wiped out by an application of the control), a discount rate of ρ=0.08 per year, and a value of 0.08 for the ratio $\rho\dfrac{C}{F}$.

For the control problem, we vary systematically σ, the volatility parameter of the pest density. For our sensitivity analysis, we cover a wide range of bioeconomic parameters values: $\mu \in \{0.05,\ 0.10,\ 0.20,\ 0.40\}$, $\omega \in \{0.00,\ 0.01,\ 0.10,\ 0.25\}$, $\rho \in \{4\%,\ 8\%,\ 12\%\}$, and $\rho\dfrac{C}{F} \in \{0.016, 0.08, 0.4\}$.

For the bioeconomic investigation problem, we set σ^2 to 0.08 per year, a to $\ln(\rho)$, and b to 1. We then generate results for a wide range of values of s, and conduct a sensitivity analysis

17

with $a \in \{\ln(\rho/3), \ln(\rho/2), \ln(\rho), \ln(2\rho)\}$ and $b \in \{0.5, 1.0, 2.0, 4.0\}$. Results were obtained using MathCad on a PC.

4.2 Results

Figures 1-3 illustrate our results for the control problem. Figure 1 shows x^* versus σ given different values of the pest density trend (μ). First, we see x^* increases with σ—if the pest population is more volatile, the optimal control threshold increases. A very volatile pest population is more likely to drop off on its own, which makes control less necessary. Since X follows a GBM, an application of Ito's lemma (see Karlin and Taylor 1981) to $Y \equiv Ln(X)$ shows it follows the Brownian motion $dY = \left(\mu - \dfrac{\sigma^2}{2} \right) dt + \sigma dz$, which has a negative downward trend for σ large enough. Figure 2 confirms this intuition—we observe the normalized value function V_c/C decreases with the volatility parameter, σ.

As expected for any given σ, Figure 1 shows x^* decreases with the trend μ–the faster a damaging pest tends to grow, the sooner control should be applied. In contrast, V_c/C increases with μ because a fast growing pest population creates more damages and requires more frequent treatments (see Figure 2).

Intuition points to control efficacy as a key parameter. Our results, however, show ω has almost no impact on x^* for the range of parameters we consider (these results are not graphed herein). The manager has no need to worry about how efficacy affects her decision to control the exotic pest. A more effective control measure, however, eliminates more of the pest and requires less frequent applications. This clearly impacts the value function, as shown on a graph of V_c/C.

versus ω (Figure 3): we see a 50 percent difference between ω=0.0 and ω=0.25 for low levels of volatility, although this difference shrinks when σ increases.[14]

Now consider the bioeconomic investigation problem. Figures 4-6 illustrate some representative results. We focus on the impacts on B^* and on V_b/C from changes in a and b—the two parameters of the distribution of time to obtain reasonable bioeconomic parameters as a function of the level of expenditure (B).

Figure 4 shows B^* as a function of s for different values of a. An increase in a implies a longer minimum time before any action can be undertaken to reduce the density of the exotic pest, and larger initial damages. We expect the optimal bioeconomic expenditure (B^*) to increase with a to decrease potentially higher damages, and this is what we observe. In addition, B^* increases with s. This occurs because more uncertainty in the duration of the bioeconomic investigation increases expected pest damages. But this increase is mild for lower values of s; it becomes quite steep, however, for larger values of s and a. This pattern reflects the exponential increase in pest density (and in pest damages) that results from our assumption that X follows a GBM. This occurs because our choice of μ leads to unbounded pest damages without control. Plots of the value function V_b normalized by control costs versus s for different values of a show similar results (not shown here).

Figures 5 and 6 illustrate how B^* and V_b/C vary as functions of s for different values of b. First, we see B^* is an increasing function of s: as for parameter a, more uncertainty in the duration of the bioeconomic investigations means higher expected pest damages initially, so more should be spent to speed up these investigations, resulting in higher values of V_b/C. As for Figure 4, the impact of s increases sharply with its value. Second, a higher value of b requires

larger expenditures to reduce the duration of bioeconomic investigations, so both B^* and V_b/C increase with b. For the range of parameters considered, B^* appears more sensitive to the value of b than to the value of a. This suggests more efforts should be devoted to understanding b, which relates directly to how spending affects the duration of bioeconomic investigations.

V. Conclusions

As the prominent biologist Daniel Simberloff (1999, p. 39) has noted: "No one has enough money or time to attempt to deal with all NIS [non-indigenous species]; there is a danger of management grinding to a halt if we attempt to exclude, eradicate, and control every alien species....So we need to be able to do some version of a risk assessment on each, even if it is a very cursory one....We have to make a decision based on the results of the risk assessment and information on the costs and likelihood of success of potential management procedures."

Simberloff's quote illustrates the realities confronting policymakers in charge of controlling exotic species. Three critical realities deserve attention. First, they face tight budget constraints. Second, not every invasive species causes damages to such a degree as to warrant immediate attention. The impacts of uncertainty in pest evolution and irreversibility in pest damages and in control costs need to be captured. Third, key bioeconomic data characterizing new exotic pests may be lacking. Sometimes it pays to wait—both to save control costs and to gather more bioeconomic data. The key questions are when to control and when to wait; and how much to invest in bioeconomic investigations.

Our proposed real options framework starts answering these questions for an exotic pest whose density follows a diffusion. For the control problem, our first order necessary condition depends on the elasticity of a discount factor, and on the elasticity of control costs plus the flow

of between-control damages; it provides a rule of thumb for when to act. We also provide a framework for finding the optimum level of effort for bioeconomic investigations. When we apply these models to an exotic pest whose density follows a GBM and control costs are fixed, we find an explicit solution for the control problem. A numerical illustration for a wide range of bioeconomic parameters shows that the efficacy of the control measure has little impact on the decision to control and it highlights the importance of investing in bioeconomic investigations under uncertainty. Our results show it is generally important to account for uncertainty in the evolution of the pest population and in the duration of bioeconomic investigations, as well as for the irreversibility of pest damages and control measures.

Future work could focus on the optimum resources to spend on detection and prevention; learning bioeconomic parameters in a Bayesian framework; and jointly managing the arrival of many exotic species. Applying our proposed framework to a case study also seems worthwhile.

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Appendix

We now show how to calculate key expressions that intervene in our formulation based on Chapter 15 in Karlin and Taylor (1981).

- Calculation of $\Gamma^A(x\,|\,y)$. For x given (with $0 < y < x$), let $R(y) = \Gamma^A(x\,|\,y)$. Karlin and Taylor (page 203-204; see (3.39) and (3.40)) show that $R(y)$ verifies:

$$\frac{\sigma^2(y)}{2}R''(y) + \mu(y)R'(y) - \rho R(y) + F(y) = 0, \tag{A.1}$$

with boundary conditions

$$R(0) = 0 \text{ and } R(x) = 0, \tag{A.2}$$

since X is absorbed at 0.

- $\Gamma^R(x\,|\,y)$ can be derived almost the same way. For x and $\varepsilon{>}0$ given (with $\varepsilon < y < x$), let this time $R(y) = \Gamma^R(x\,|\,y)$. As for $\Gamma^A(x\,|\,y)$, $R(y)$ verifies (A.1), but this time the boundary conditions are (see Dixit 1993 for the first one)

$$R'(\varepsilon){=}0 \text{ and } R(x) = 0. \tag{A.3}$$

The same logic for boundary conditions applies to the calculation of the expected discount factors.

- Expression of $D^A_{0,x|y}$. Let $W(y) \equiv D^A_{0,x|y} == E\left(e^{-\rho T^A_{0,x|y}} \right)$. From Karlin and Taylor (page 204; see (3.41) and (3.42)), $W(y)$ verifies the linear, second-order, ordinary differential equation

$$\frac{\sigma^2(y)}{2}\frac{d^2W(y)}{dy^2} + \mu(y)\frac{dW(y)}{dy} - \rho W(y) = 0. \tag{A.4}$$

The boundary conditions here are

$$W(0) = 0 \text{ and } W(x) = 1. \tag{A.5}$$

The constraint $W(x)=1$ is obvious here. The requirement $W(0)=0$ reflects the fact that X is absorbed at 0: indeed, the only possible value for $T^A_{0,x|y}$, which is the first time X hits x starting from *y conditional on X hitting x before 0*, is $+\infty$.

- Finally, let us derive $D^R_{\varepsilon,x|y}$. Consider again x and $\varepsilon{>}0$ given (with $\varepsilon < y < x$), but this time $W(y)$ refer to $D^R_{\varepsilon,x|y}$. As above, $W(y)$ verifies (A.4), but the boundary conditions become (see Dixit 1993 for the first one)

$$W'(\varepsilon) = 1 \text{ and } W(x) = 1, \tag{A.6}$$
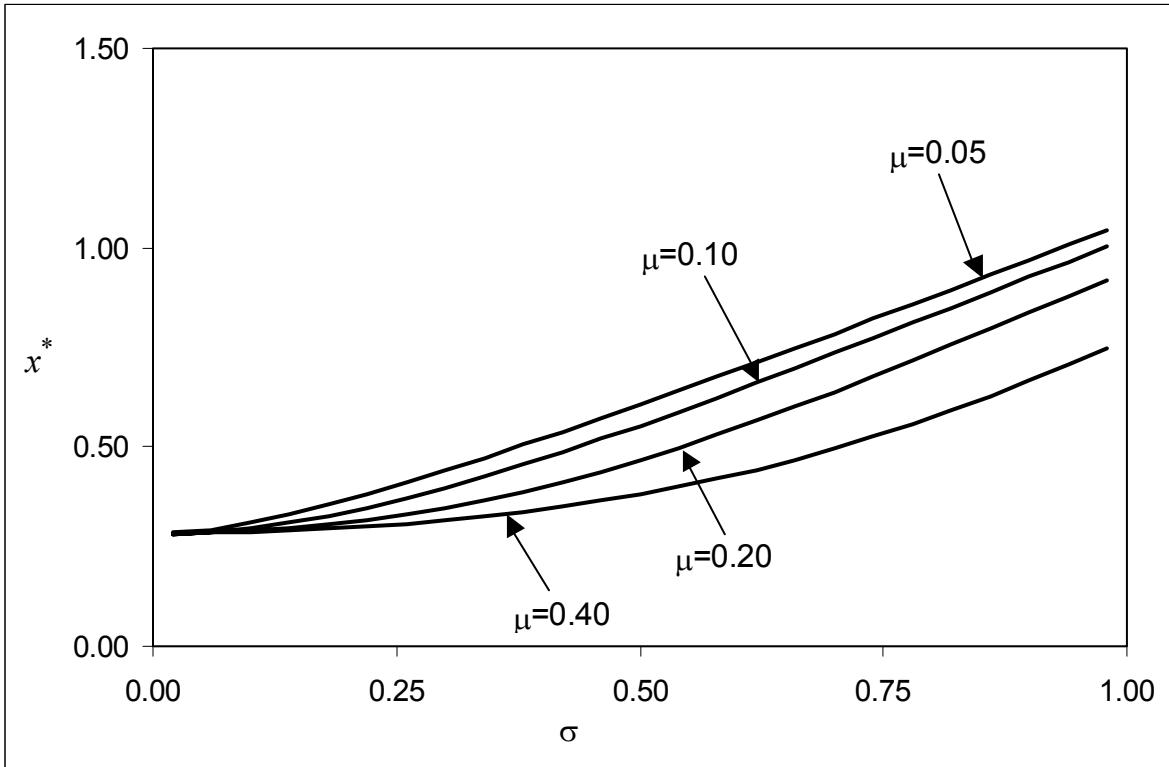
because X is reflected at ε.

Figure 1: x^* versus σ for different values of μ.

Notes. These results were generated with $\delta=2$, $\rho=0.08$ per year, $\rho C/F=0.08$, $\omega=0.01$. μ and σ are respectively the trend and the volatility parameters for the pest population; δ is the damage exponent and F is a scaling factor for the pest damage function; ρ is the annual effective discount rate; C is the cost of treating the area under consideration; ω is the fraction of the pest population remaining after treatment; finally, x^* is the action threshold for controlling the pest.
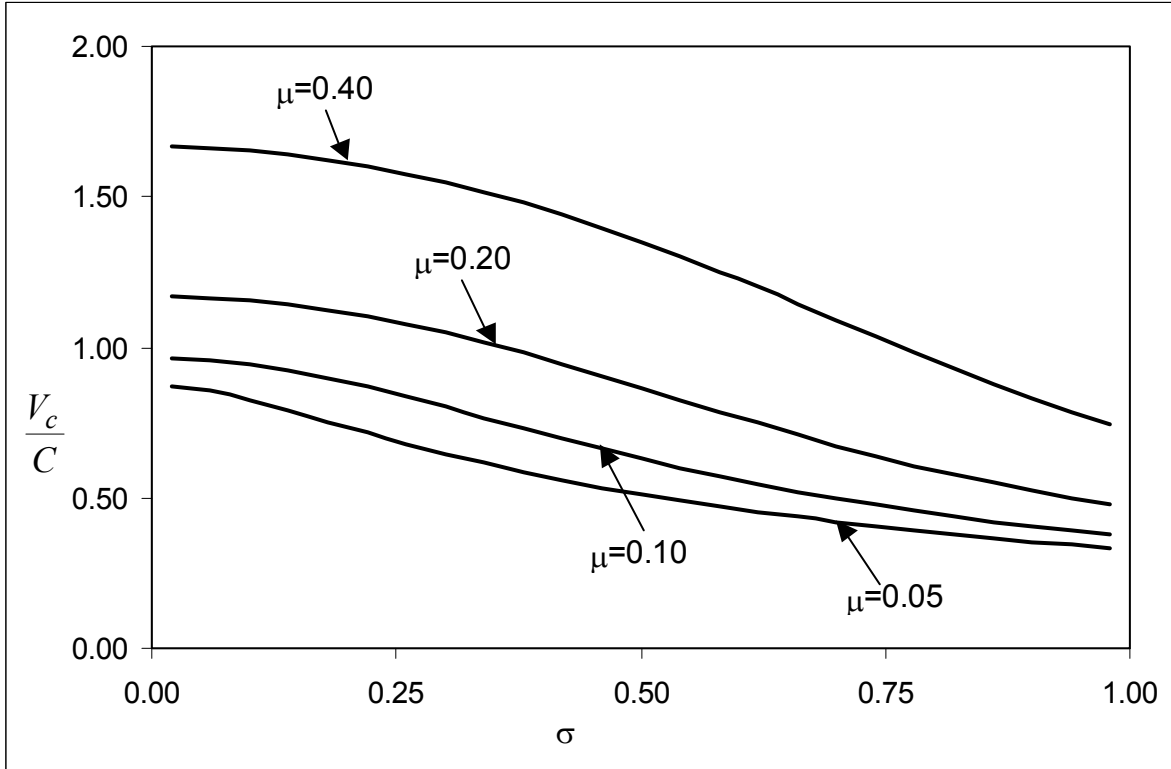
Figure 2: V_c/C versus σ for different values of μ.

Notes. These results were generated with $\delta=2$, $\rho=0.08$ per year, $\rho C/F=0.08$, $\omega=0.01$. μ and σ are respectively the trend and the volatility parameters for the pest population; δ is the damage exponent and F is a scaling factor for the pest damage function; ρ is the annual effective discount rate; C is the cost of treating the area under consideration and V_c is the value function for the control problem; finally ω is the fraction of the pest population remaining after treatment.
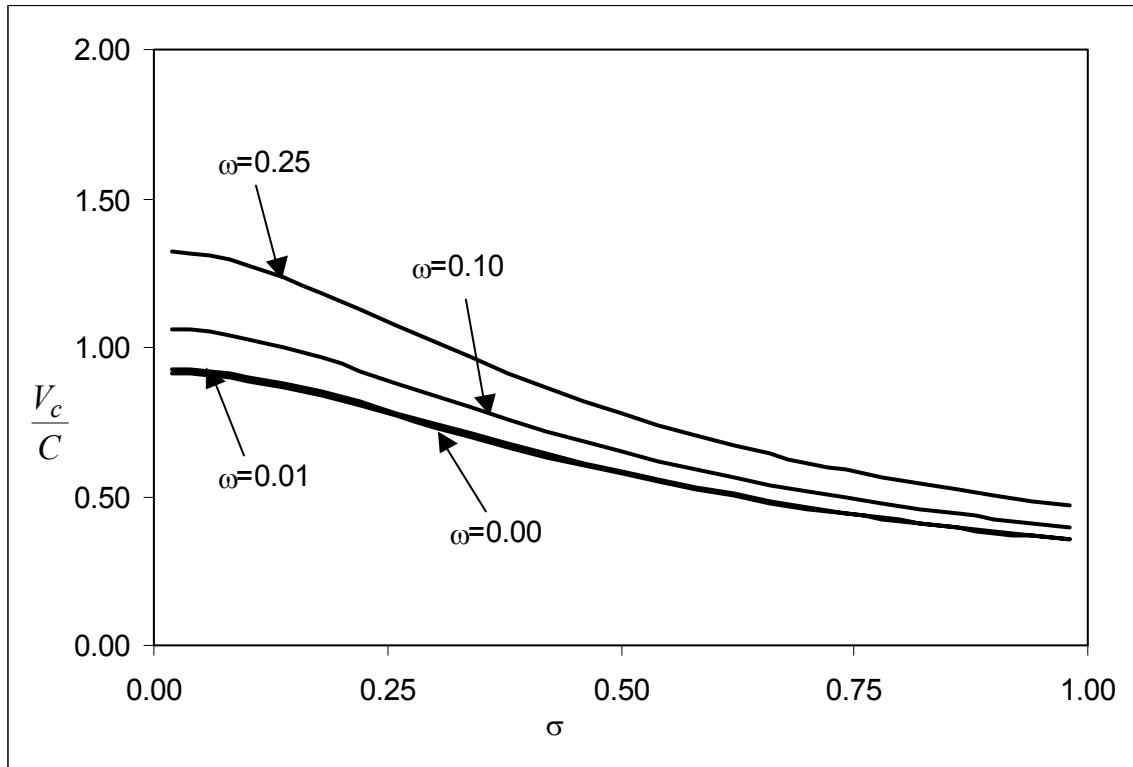
Figure 3: V_c/C versus σ for different values of ω.

Notes. These results were generated with $\mu=0.08$ per year, $\delta=2$, $\rho=0.08$ per year, and $\rho C/F=0.08$.

μ and σ are respectively the trend and the volatility parameters for the pest population; δ is the

damage exponent and F is a scaling factor for the pest damage function; ρ is the annual effective

discount rate; C is the cost of treating the area under consideration and V_c is the value function

for the control problem; finally ω is the fraction of the pest population remaining after treatment
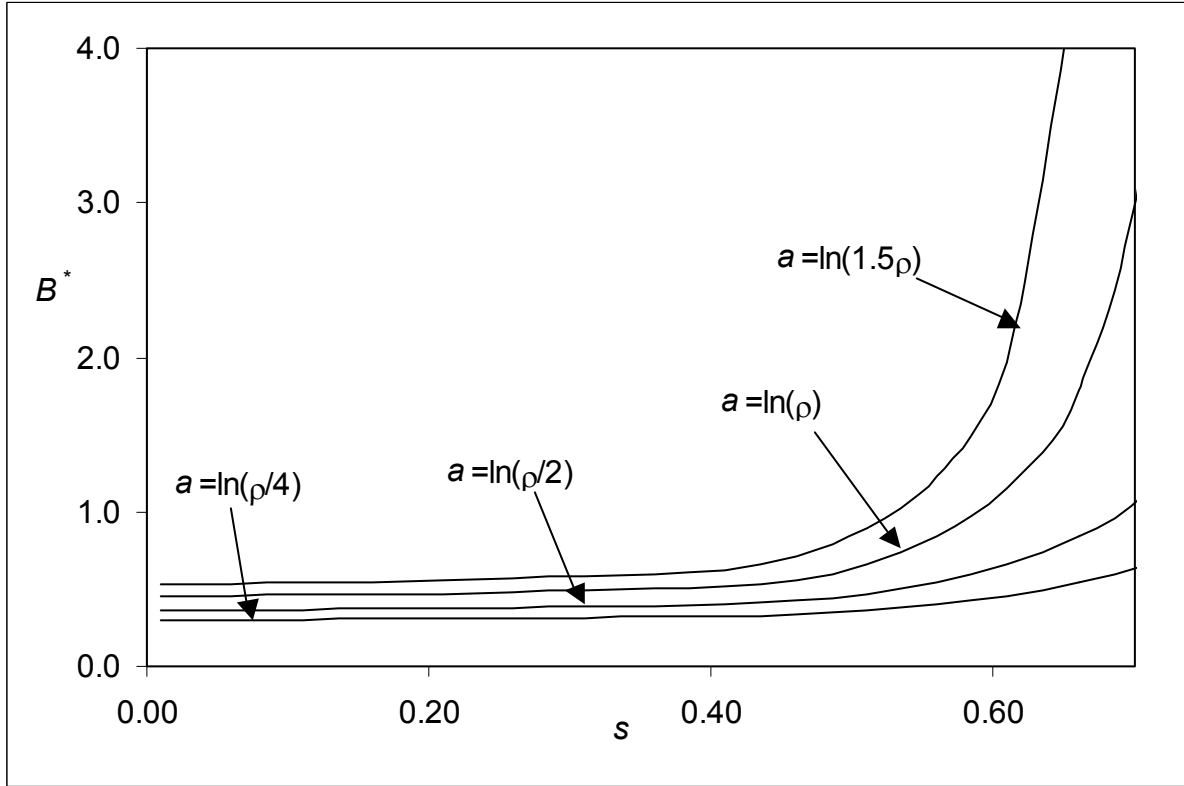
(the efficacy of the control measure).

Figure 4: B^* versus s for different values of a.

Notes. These results were generated with $\mu=0.08$ per year, $\sigma^2=0.08$ per year, $\delta=2$, $\omega=0.01$, $\rho=0.08$ per year, $\rho C/F=0.08$, and $b=1$. As before, μ and σ are respectively the trend and the volatility parameters for the pest population; δ is the damage exponent and F is a scaling factor for the pest damage function; ρ is the annual effective discount rate; C is the cost of treating the area under consideration; and ω is the fraction of the pest population remaining after treatment. If T is the time to find satisfactory bioeconomic parameters, $\ln(\rho T)$ is normally distributed with mean $a + bB^{-1}$ and standard deviation s, where B is how much the resource manager spends on bioeconomic investigations. B^* is the optimal value of B.
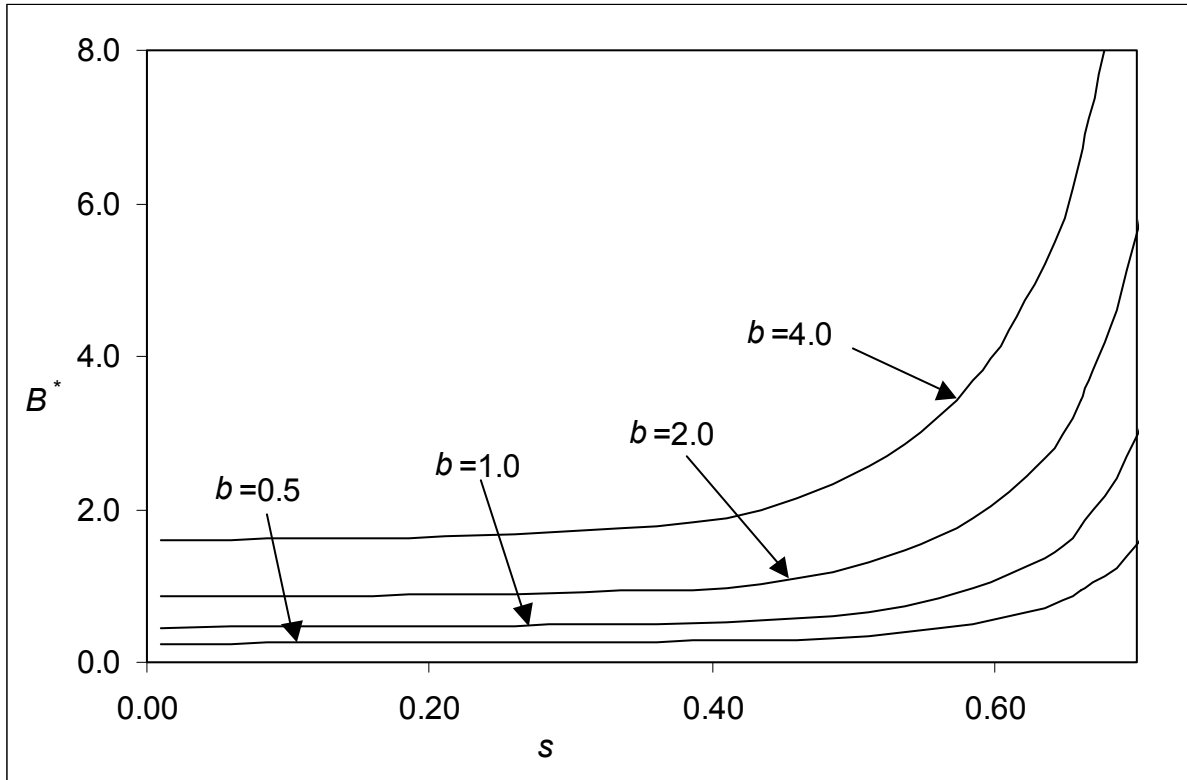
Figure 5: B^* versus s for different values of b.

Notes. These results were generated with μ=0.08 per year, σ^2=0.08 per year, δ=2, ω =0.01, ρ=0.08 per year, $\rho C/F$=0.08, and a=ln(ρ). μ and σ are respectively the trend and the volatility parameters for the pest population; δ is the damage exponent and F is a scaling factor for the pest damage function; ρ is the annual effective discount rate; C is the cost of treating the area under consideration; and ω is the fraction of the pest population remaining after treatment. If T is the time to find satisfactory bioeconomic parameters, $\ln(\rho T)$ is normally distributed with mean $a + bB^{-1}$ and standard deviation s, where B is how much the resource manager spends on bioeconomic investigations. B^* is the optimal value of B.
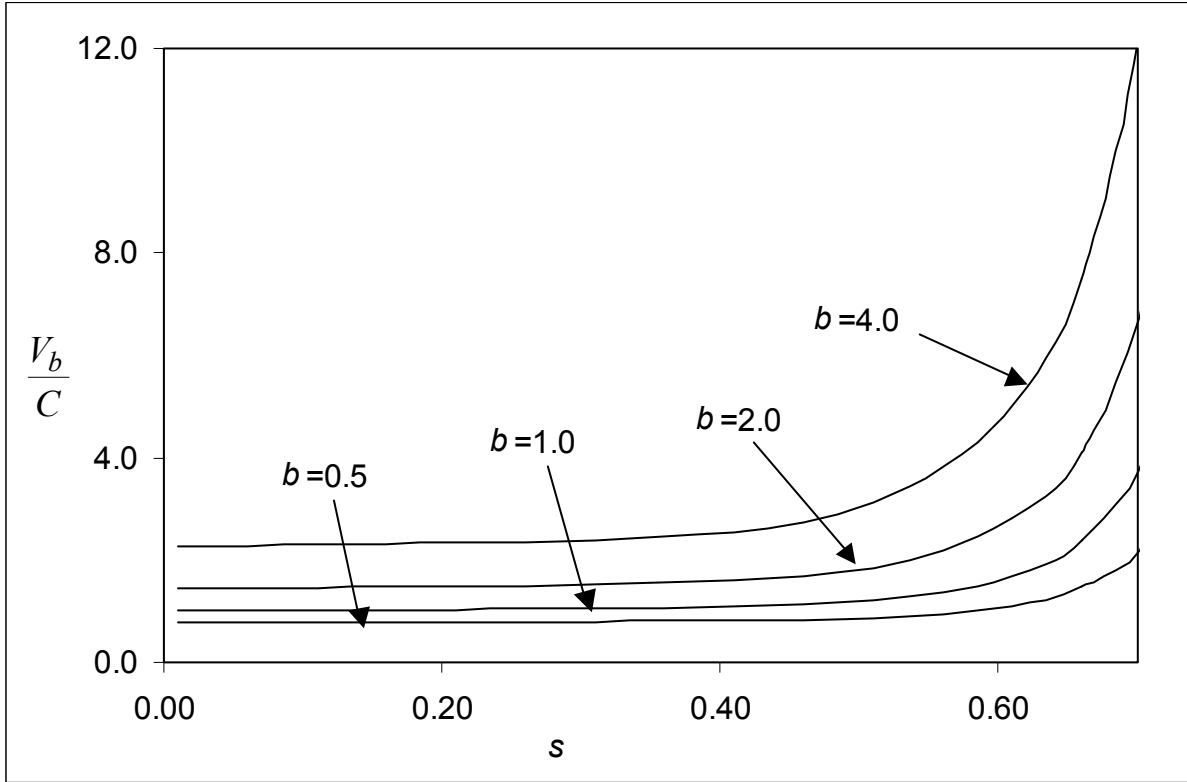
Figure 6: V_b/C versus s for different values of b.

Notes. These results were generated with μ=0.08 per year, σ^2=0.08 per year, δ=2, ω=0.01, ρ=0.08 per year, $\rho C/F$=0.08, and $a=\ln(\rho)$. μ and σ are respectively the trend and the volatility parameters for the pest population; δ is the damage exponent and F is a scaling factor for the pest damage function; ρ is the annual effective discount rate; C is the cost of treating the area under consideration; and ω is the fraction of the pest population remaining after treatment. If T is the time to find satisfactory bioeconomic parameters, $\ln(\rho T)$ is normally distributed with mean $a + bB^{-1}$ and standard deviation s, where B is how much the resource manager spends on bioeconomic investigations. V_b is the value function for the biological investigation problem.

[1] See Conrad (2000) or Saphores (2000, 2003) for applications of option-pricing theory to natural resource issues.

[2] We simplify our formulation by assuming there are no minimum delays between two successive applications of the control measure.

[3] Assuming that the pest population follows (1) is a general and convenient way of modeling uncertainty in continuous time. Diffusions have been widely used in economics, biology, or physics; for examples, see Karlin and Taylor (1981).

[4] Following the literature, X denotes the random variable for the pest density and x denotes one of the values it can take.

[5] If the resource manager faces a very fast growing pest that causes large damages, and if only a relatively inefficient control measure is available, it may be necessary to apply this control measure repeatedly to bring the pest density between 0 and x^*.

[6] As pointed out by a referee, assuming that X follows a GBM implies that the pest population tends to explode exponentially; it may therefore overestimate pest damages if the pest population is quickly limited by the availability of food or the action of predators.

[7] We consider a lower barrier and take it to 0.

[8] The second order condition at $x=y=x^*$ is $\dfrac{\partial^2 V_c}{\partial x^2}\big|_{x=y=x^*} = \dfrac{\theta C \delta}{x^{*2}(1-\omega^{\theta})} > 0$, so x^* is a minimum.

[9] We assume x is divided by a unit constant with the same unit as X in (10), so F is expressed in $ per unit of time.

[10] We have $\dfrac{dx^*}{d\mu} = \dfrac{\partial x^*}{\partial\theta}\dfrac{d\theta}{d\mu} + \dfrac{\partial x^*}{\partial\kappa}\dfrac{d\kappa}{d\mu}$ with $\dfrac{d\theta}{d\mu} = \dfrac{-2\theta}{2\sigma^2\theta + 2\mu - \sigma^2}$, $\dfrac{d\kappa}{d\mu} = -\dfrac{\delta}{\rho}$, $\dfrac{\partial x^*}{\partial\theta} = \dfrac{-x^*}{\theta(\theta-\delta)}$,

and $\dfrac{\partial x^*}{\partial\kappa} = \dfrac{x^*}{\delta\kappa}$. The equivalence $(\kappa < 0) \Leftrightarrow (\theta - \delta < 0)$ results from the definitions of θ and κ.

[11] As a result, a is dimensionless and b has the same units as B. Ln(T) is normally distributed with mean $a+bB^{-1}$-ln(ρ) and standard deviation s.

[12] The magnitude of a normalized value function does not depend on the units selected.

[13] Derivations are available from the authors upon request.

[14] We conducted but do not report other numerical results that confirm our discussion of the unambiguous sensitivity analysis for expression (17).