Adaptive Learning and Inflation Persistence

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Abstract

What generates persistence in inflation? Is inflation persistence structural?

This paper investigates learning as a potential source of persistence in inflation. The paper focuses on the price-setting problem of firms and presents a model that nests structural sources of persistence (indexation) and learning. Indexation is typically necessary under rational expectations to match the inertia in the data and to improve the fit of estimated New Keynesian Phillips curves.

The empirical results show that when learning replaces the assumption of fully rational expectations, structural sources of persistence in inflation, such as indexation, become unsupported by the data. The results suggest learning behavior as the main source of persistence in inflation. This finding has implications for the optimal monetary policy.

The paper also shows how one's results can heavily depend on the assumed learning speed. The estimated persistence and the model fit, in fact, vary across the whole range of constant gain values. The paper derives the best-fitting constant gains in the sample and shows that the learning speed has substantially changed over time.

Keywords: adaptive learning, inflation persistence, sticky prices, best-fitting constant gain, learning speed, expectations.

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1. INTRODUCTION

What creates **persistence** in inflation? Is inflation persistence a **structural** characteristic of industrialized economies? Or does persistence instead vary with the monetary policy regime, for example?

Despite several studies concerning inflation dynamics, economists have not reached a consensus about the answers to the previous questions. This paper aims to address some of those questions, proposing and evaluating learning as a possible solution.

A vast literature uses sticky price models to describe inflation behavior. The need to match the sluggishness of price movements and to allow for some real effects of monetary policy led many researchers to abandon the hypothesis of flexible prices. Current dynamic general equilibrium models often incorporate price stickiness and imperfect competition. These frameworks are built from the optimizing choices of economic agents and are therefore theoretically appealing. By incorporating various rigidities, they were also expected to be empirically more realistic.

But sticky price models still fail to imply realistic levels of inflation persistence. In the baseline New Keynesian model, at least, inflation is in fact an entirely forward-looking variable and all of its inertia is inherited from the inertia of an exogenous driving variable, i.e. real marginal costs or the output gap. To improve the empirical fit of these models, it is necessary to extend them by introducing additional sources of persistence. Those extensions allow researchers to introduce the dependence of current inflation on *lagged inflation*. The additional channels of inertia have been variously modeled in the literature by incorporating rule-of-thumb behavior, quadratic adjustment costs or indexation to past inflation. The impulse responses become more in line with those derived from VAR models, implying adjustment delays and sluggish responses.

Gali' and Gertler (1999), for example, allow for the existence of a fraction of firms that deviate from full rationality and set instead their prices using simple rule-of-thumbs. Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003), Giannoni and Woodford (2003) and Woodford (2003), on the other hand, allow partial or full indexation of prices to past inflation rates for firms not adjusting their prices optimally in a given period, as an extension to the standard Calvo (1983) pricing model. The implications are similar: current inflation ceases to be a merely forward-looking variable, now also depending on lagged inflation. Those variations improve the empirical properties of their models.

Indeed, it seems that inflation can be well represented by a specification that nests both forward- and backward-looking terms. But the relative importance of the two components is a matter of dispute.

Gali' and Gertler (1999) propose what they call the "New Hybrid Phillips Curve", in which real marginal costs are the main driving variable of inflation, and both expected and past inflation affect the dynamics of current inflation. They argue that inflation is mainly a forward-looking phenomenon, finding roughly 2/3 of rational and 1/3 of rule-of-thumb price setters from their GMM estimation. Opposite is the view of Fuhrer and Moore (1995) and Fuhrer (1997), who, conversely, obtain inflation as purely backward-looking. The opinions in the middle are innumerable.

Together with the sources of persistence in inflation, another key issue lies in understanding whether inflation persistence is an intrinsic characteristic of industrialized economies.[1] Recent studies trying to shed light on this issue are Cogley and Sargent (2005), who find evidence of a structural break in inflation dynamics, Levin and Piger (2002), who analyzing a panel of industrial countries conclude that high inertia is not an inherent characteristic of industrial economies, and Erceg and Levin (2003), who also oppose the view of inflation persistence as structural and argue that it can depend on central bank's perceived credibility. Reis and Pivetta (2003), on the other hand, find that inflation persistence has remained high and substantially unchanged since 1965.

This paper tries to take a different route, by suggesting a plausible different source of inflation inertia. The paper highlights the potential for **adaptive learning** in creating persistence in inflation and tries to evaluate empirically the importance of learning.

Following a growing literature on learning in macroeconomics, I model private agents as econometricians, who estimate simple models of the economy and form expectations from them. As agents obtain more data over time, they update their parameter estimates through Constant-Gain Learning (CGL). This represents a way to model the learning process of agents concerned about potential structural breaks in the economic relationship they need to learn. One possible way to proceed to evaluate the empirical importance of learning would consist of simulating the economy with non-fully rational expectations and assess if those are able to generate enough persistence compared to the data. This paper instead proposes a different experiment: I develop an optimizing model where I introduce learning, but I also allow for a structural characteristic that induces persistence in inflation. In fact, I allow for indexation to past inflation by non-optimizing firms. The model therefore *nests* two potential sources of persistence in inflation: learning and indexation.

It becomes an empirical matter to understand whether structural sources of persistence remain essential to match the data, as they were under rational expectations, when learning is instead introduced.

A related work is Ball (2000), who similarly focuses on expectations and drops the assumption of full rationality. Ball allows agents to use optimal univariate forecast rules as an alternative to rational or purely adaptive expectations. The current paper, instead, introduces learning by agents. Also related are the papers by Roberts (1997, 1998), and Adam and Padula (2003), who estimate inflation equations using subjective expectations from surveys. But this paper provides in fact a way to model the formation of those subjective expectations.

There are other attempts to enrich the models to imply more inertial dynamics. Dotsey and King (2001), Guerrieri (2005), Holden and Driscoll (2003), Coenen and Levin (2004) propose alternative adjustments to the model, all with the scope of generating additional persistence, without focusing on expectations. Mankiw and Reis (2002) and Woodford (2003a) are recent influential studies trying to explain inertia by agents' limited ability to update or absorb information.

This paper therefore aims to contribute to the large literature on inflation dynamics, proposing and evaluating a different explanation of its persistence. Moreover, the current paper can also be seen as a contribution to the growing literature on adaptive learning.

The majority of studies in the previous adaptive learning literature, in fact, has been mainly interested in studying the convergence of models with learning to the rational expectations equilibrium. This line of research is comprehensively surveyed in Evans and Honkapohja (2001). Similar scope have the applications in monetary policy models, such as Bullard and Mitra (2002), and Evans and Honkapohja (2003).

Recently, this area of research has expanded its objectives, applying learning to explain U.S. inflation in the 1970s (Sargent 1999, Orphanides and Williams 2003, Bullard and Eusepi 2005, Primiceri 2003). Williams (2003) examines instead the empirical importance of adaptive learning in a business cycle model. This paper shares the interest in these new objectives and it aims to contribute to the understanding of the empirical implications of learning.

By estimating a model with deviations from rational expectations, this paper represents a simple example of what Ireland (2003) has defined "Irrational Expectations Econometrics". Ireland pointed out the results obtained by the theoretical literature on learning and emphasized the need for an "Irrational Expectations Econometrics" that would complement those results, by assessing the empirical importance of learning. This is what the paper tries to do. This paper focuses on a single equation estimation, while a companion paper, Milani (2004b), pursues a joint estimation of a full New-Keynesian macro-model with learning by likelihood-based Bayesian methods.

The paper focuses on the inflation equation. The aim is to see how much persistence can be explained by learning. I discover this by looking at the estimates for the degree of indexation to past inflation, necessary to match the data. In models with rational expectations, the coefficient on indexation has been fixed (CEE) or estimated (Boivin and Giannoni (2003) and Giannoni and Woodford (2003)) to be equal to 1, or to typically large values.

The empirical results highlight the unimportance of forms of structural persistence in inflation. When I drop the assumption of rational expectations, allowing instead economic agents to form their expectations through constant-gain learning, the estimates of the degree of indexation to lagged inflation fall to 0. This suggests that learning can account for a sizeable amount of persistence in inflation.

As a related issue, I recognize that when modeling learning behavior, researchers dispose of a number of degrees of freedom. An important choice is the learning speed of private agents, i.e. the value to assign to the constant gain parameter. The results can dramatically change for different assumptions about the learning speed. This is certainly a concern, also in my context. This concern leads me to perform an additional experiment: I examine the relationship between the implied estimates of structural persistence and the possible learning speeds. The estimates vary a great deal across a large range of gain values. It becomes therefore necessary to compare the different possible gains and I will try to shed some light on this by comparing the fit under the various gain coefficients. In this way, I obtain the value of the best-fitting constant gain coefficient in the sample. This is close to values typically assumed (without estimation) in the previous learning literature. I also consider the evolution of the gain coefficient over time and I find that the best-fitting constant gain has been much larger in the post-1982 sample compared with the best-fitting gain in the pre-1979 sample, indicating faster learning in the latest two decades.

Although several papers have started to employ constant-gain learning, estimates of the gain lack in the literature. This paper contributes to the literature by providing a first attempt to estimate the best-fitting constant gain in a model of inflation dynamics.

The finding that inflation persistence is not due to structural characteristics, but to learning behavior by agents carries some important policy implications. The welfare loss will be different under the alternative sources of persistence and, consequently, optimal monetary policy will be different. A successful *management of expectations,* as called for by Woodford (2003b), will be crucial under learning.

The rest of the paper is structured as follows. Section 2 presents the model, starting from the microfoundations of a dynamic optimizing general equilibrium (DGE) model under rational expectations. Section 3 presents the aggregate law of motion for inflation and describes the expectations' formation mechanism. Section 4 derives the main empirical results of the paper. Section 5 and 6 explore the relationship between learning speed and the implied estimated inflation persistence and in-sample fit. Section 7 investigates the robustness of the empirical results to alternative assumptions about the learning rule, while Section 8 discusses the policy implications of the results. Section 9 concludes.

2. THE MODEL

In this section, I derive the law of motion for inflation, which will correspond to the popular New Keynesian Phillips curve, but augmented along two directions. First, to induce a more realistic degree of inertia in inflation, I allow non-optimizing firms to update their prices through indexation to lagged inflation, as proposed by Christiano, Eichenbaum and Evans (2005). Then, the paper makes an important departure from the usual expectations formation mechanism. I drop the assumption of rational expectations and assume instead that agents behave as econometricians, estimating an economic model and from that model forming their expectations.

In what follows, I start by setting up the optimal price-setting problem for a firm under rational expectations. I shall introduce subjective (possibly non-rational) expectations and learning in the next section. The current paper focuses only on the price-setting problem by firms. A full model with consumer optimization and monetary policy is described in Milani (2004b).

2.1. The Household Problem

I consider a standard economy populated by a continuum of households indexed by i, maximizing a discounted sum of future utilities. The generic household maximizes the following intertemporal utility function:

$$E_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[U\left(C_T^i, ; \xi_T\right) - \int_0^1 v(h_T^i(j); \xi_T) dj \right] \tag{1}$$

where consumer's utility depends positively on an index of consumption C_T^i, and negatively on the amount of labor supplied for the production of good j, $h_T^i(j)$; ξ_T is an aggregate preference shock, whereas $\beta \in (0,1)$ is the usual household's discount factor. E_t^i here denotes rational (model-consistent) expectations. The consumption index is of the Dixit-Stiglitz CES form

$$C_t^i \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

and the associated aggregate price index is expressed by

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \tag{3}$$

where θ represents the elasticity of substitution between differentiated goods. From the household's problem, I just need to derive the marginal utility of real income (here equal to the marginal utility of consumption, the usual $\lambda_t = U_C(C_T^i, ; \xi_T)$), which will appear in the aggregate supply relationship. The other details of consumer optimization can therefore remain implicit in the background[2].

2.2. *Optimal Price Setting*

Let's consider the firm's problem. I assume Calvo price-setting, so that a fraction $0 < 1 - \alpha < 1$ of prices are allowed to change in a given period and are optimally set. The price of the remaining fraction α, that is not optimally fixed in the period, is adjusted according to the indexation rule

$$\log p_t(i) = \log p_{t-1}(i) + \gamma \pi_{t-1} \tag{4}$$

Similarly to Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003) and Giannoni and Woodford (2003), I allow firms to index their prices to past inflation when they cannot set their prices optimally. This extension is typically needed to improve the empirical fit of the model and it allows one to derive more realistic impulse response functions. $0 \leq \gamma \leq 1$ represents the degree of indexation to past inflation (Christiano, Eichenbaum and Evans (2005) assumed $\gamma = 1$, meaning full indexation). An alternative solution to introduce dependence on lagged inflation would consist of assuming a fraction of rule-of-thumb firms. An example is Gali' and Gertler (1999), who define the derived equation 'New Hybrid Phillips Curve'.

The demand curve for product i takes the form:

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta} \tag{5}$$

where $Y_t = \left[\int_0^1 y_t(i)^{\frac{(\theta-1)}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ is the aggregate output, P_t is the aggregate price as in (3). Each firm i has a production technology $y_t(i) = A_t f(h_t(i))$, where A_t is an exogenous technology shock, $h_t(i)$ is labor input and the function $f(\cdot)$ satisfies the usual Inada conditions.

Since each firm faces the same demand function (5), all firms allowed to change their price in period t will set the same price p_t^* that maximizes the expected present discounted value of future profits:

$$E_t^i \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_T^i \left(p_t(i) \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\gamma} \right) \right] \right\} \tag{6}$$

where $Q_{t,T} = \beta^{T-t} \frac{P_t}{P_T} \frac{U_c(Y_T,\xi_T)}{U_c(Y_t,\xi_t)}$ is the stochastic discount factor (U_c is the marginal utility of an additional unity of income), and $\Pi_T^i(\cdot)$ denotes firm's profits. Firms discount future profits at rate α, since they can expect the optimal price chosen at date t to apply in period T with probability α^{T-t}. The firm chooses $\{p_t(i)\}$ to maximize the flux of profits (6), for given $\{Y_T, P_T, w_T(j), A_T, Q_{t,T}\}$ for $T \geq t$ and $j \in [0,1]$.

The firm's problem results in the first-order condition

$$E_t^i \left\{ \begin{array}{c} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} U_c(Y_T;\xi_T) Y_T P_T^{\theta} \left(\frac{P_{T-1}}{P_{t-1}}\right)^{\gamma(1-\theta)} \cdot \\ \left[\widehat{p}_t^*(i) - \mu P_T s \left(Y_T \left(\frac{\widehat{p}_t^*(i)}{P_T}\right)^{-\theta} \left(\frac{P_{T-1}}{P_{t-1}}\right)^{-\gamma\theta}, Y_T; \xi_T \right) \right] \end{array} \right\} = 0 \tag{7}$$

where $\mu = \theta/(\theta-1)$, $s(\cdot)$ is firm i's real marginal cost function in period $T \geq t$, given price $\widehat{p}_t^*(i)$, set in t.

The Dixit-Stiglitz aggregate price index evolves according to the law of motion:

$$P_t = \left[\alpha \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma} \right)^{1-\theta} + (1-\alpha) p_t^{*1-\theta} \right]^{\frac{1}{1-\theta}} \tag{8}$$

From a log-linear approximation of the firm's first order condition[3] and some manipulations, I obtain:

$$\widehat{p}_t^* = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\frac{1-\alpha\beta}{1+\omega\theta} (\omega + \sigma^{-1}) x_T + \alpha\beta \left(\widehat{\pi}_{T+1} - \gamma \widehat{\pi}_T \right) \right] \tag{9}$$

where $\omega > 0$ is the elasticity of firm i's real marginal cost function $s(\cdot)$ with respect to its output $y_t(i)$, $\sigma \equiv -U_c/(U_{cc}\overline{C})$ is the usual intertemporal elasticity of substitution, and '$\hat{}$' denotes log deviations from the steady state[4].

From a log-linear approximation of the aggregate price index, notice that $\widehat{p}_t^* = \frac{\alpha}{(1-\alpha)}\left(\pi_t - \gamma\pi_{t-1}\right)$, which plugged in the previous expression gives:

$$\widetilde{\pi}_t = \delta x_t + \widehat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\delta\alpha\beta x_{T+1} + (1-\alpha)\,\beta\widetilde{\pi}_{T+1}\right] \tag{10}$$

where

$$\widetilde{\pi}_t = \pi_t - \gamma\pi_{t-1} \tag{11}$$

$$\delta \equiv \frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\frac{\left(\omega+\sigma^{-1}\right)}{(1+\omega\theta)} \equiv \frac{(1-\alpha)\,(1-\alpha\beta)}{\alpha}\zeta > 0 \tag{12}$$

and $\zeta \equiv \frac{\left(\omega+\sigma^{-1}\right)}{1+\omega\theta}$. Equation (10) can be quasi-differenced to obtain a relationship between current and one-period-ahead expected inflation

$$\pi_t - \gamma\pi_{t-1} = \delta x_t + \beta E_t\left[\pi_{t+1} - \gamma\pi_t\right] + u_t \tag{13}$$

where I have added an exogenous cost-push shock u_t. A recent strand of literature (Gali' and Gertler (1999), Sbordone (2003)) suggests using the real marginal cost as the driving variable for inflation, instead of the more commonly used output gap. As discussed in Woodford (2003b), the relationship between inflation and marginal costs holds under weaker assumptions. In fact, when the marginal cost replaces the output gap there is no need to assume any specific theory of wage-setting, for example.

Therefore, the relationship (13) can be re-expressed in terms of the real marginal cost s_t:

$$\pi_t - \gamma\pi_{t-1} = \frac{(1-\beta\alpha)(1-\alpha)}{\alpha} s_t + \beta E_t\left[\pi_{t+1} - \gamma\pi_t\right] + u_t \tag{14}$$

Notice that I could have derived similar equations for inflation dynamics, only with different restrictions on the parameters, assuming the existence of some rule-of-thumb firms (Gali' and Gertler (1999), Amato and Laubach (2003)), instead of indexation. The results in the following of the paper are not dependent on this choice.

3. INFLATION DYNAMICS WITH LEARNING

The aggregate dynamics for inflation in the model is given by eq. (13). I now relax the strong informational assumptions characterizing firms' knowledge under rational expectations. In this section, I assume that firms have subjective (and possibly non-rational) expectations. Subjective expectations are denoted by $\widehat{E}_t$.[5] The law of motion for inflation under subjective expectations becomes

$$\pi_t - \gamma\pi_{t-1} = \delta x_t + \beta \widehat{E}_t \left[\pi_{t+1} - \gamma\pi_t\right] + u_t \tag{15}$$

This can be re-expressed as

$$\pi_t = \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{\beta}{1+\beta\gamma}\widehat{E}_t\pi_{t+1} + \frac{\delta}{1+\beta\gamma}x_t + u_t \tag{16}$$

Under this specification, firms need to forecast future inflation rates to determine current inflation. The next paragraph will give some details about how agents form such forecasts.

3.1. Expectations Formation: Adaptive Learning

Firms do not know the correct model of inflation dynamics. They behave as econometricians, estimating an economic model and forming expectations from that model. For simplicity, I start assuming that firms estimate a simple linear univariate AR(1) model to form their forecasts of inflation:

$$\pi_t = \phi_{0,t} + \phi_{1,t}\pi_{t-1} + \varepsilon_t \tag{17}$$

In the estimation, they exploit the entire history of available data up to period t, $\{1, \pi_{t-1}\}_0^{t-1}$. Eq. (17) is called the **"Perceived Law Motion"** or **PLM** of the agents. Notice that, although in the estimation I will use demeaned variables, I recognize that agents need to estimate an intercept as well as slope parameters. A strictly positive intercept on inflation would signal that agents expect a positive target for inflation in the sample. In the next sections, I shall evaluate the robustness of the empirical results to different PLMs.

As new data become available, agents update their estimates according to the Constant-Gain learning (CLG) formula

$$\begin{aligned} \widehat{\phi}_t &= \widehat{\phi}_{t-1} + \kappa R_{t-1}^{-1} X_t (\pi_t - X_t' \widehat{\phi}_{t-1}) && (18) \\ R_t &= R_{t-1} + \kappa (X_{t-1} X_{t-1}' - R_{t-1}) && (19) \end{aligned}$$

where the first expression describes the updating of the forecasting rule coefficients $\widehat{\phi}_t = \left(\phi_{0,t}, \phi_{1,t}\right)'$ over time, and the second shows the evolution of R_t, the matrix of second moments of the stacked regressors $X_t \equiv \{1, \pi_{t-1}\}_0^{t-1}$. The constant gain is expressed by parameter κ and, compared with the recursive least squares (RLS) gain (equal to t^{-1}), it represents a simple way to model learning of an agent concerned about potential structural breaks at unknown dates. Constant-gain learning has also been defined as 'perpetual' learning, since learning will take place forever and the system will not converge to the RE solution (but at most to a stochastic distribution around it). A larger κ would imply faster learning of structural breaks, but it would also lead to higher volatility around the steady state. An appealing feature of this framework is that it embeds the standard rational expectations hypothesis as a special limiting case, i.e. the case for the gain coefficient κ converging to 0. An increasing number of papers has used constant-gain learning (Orphanides and Williams 2003, 2004, Primiceri 2003).

I assume that agents, when forming expectations in period t, have access to information only up to $t-1$: I therefore replace $\widehat{E}_t$ with $\widehat{E}_{t-1}$. Using their PLM and the updated parameter estimates $\widehat{\phi}_t$, agents form expectations for $t+1$ as

$$\widehat{E}_{t-1}\pi_{t+1} = \phi_{0,t-1}\left(1+\phi_{1,t-1}\right) + \phi_{1,t-1}^2\pi_{t-1} \tag{20}$$

To summarize, the model of the economy is composed by the inflation dynamics equation (16) (Phillips curve), agents' beliefs (17), updating equations (18), (19), and the forecasting rule (20).

Substituting agents' expectations formed from their PLM as in (20) into the inflation dynamics equation (16), I derive the **"Actual Law of Motion"** or **ALM** of the economy, i.e. the law of motion of π_t for a *given* PLM:

$$\pi_t = \frac{\beta\phi_{0,t}\left(1+\phi_{1,t}\right)}{1+\beta\gamma} + \frac{\gamma+\beta\phi_{1,t}^2}{1+\beta\gamma}\pi_{t-1} + \frac{\kappa}{1+\beta\gamma}x_t + u_t \tag{21}$$

where the reduced-form coefficients are time-varying and are convolutions of the structural parameters describing inflation dynamics and of the coefficients representing agents' beliefs. In models with adaptive learning, it is commonly assumed that, in each period t, agents use an econometric model to form their expectations about future inflation, but they do not take into account their subsequent updating in periods $T > t$. Therefore, they act as adaptive decision-makers, in accordance with what Kreps (1998) defines as an *anticipated utility model*[6].

Obviously, although there is just one form of full rationality, several alternative ways to model learning are possible. The paper considers a simple learning rule (assuming that agents use univariate autoregressive models). But it is worth pointing out that such a simple mechanism of expectations formation fits quite well with inflation expectations from surveys, such as the expected inflation series from the Survey of Professional Forecasters, for example (Branch and Evans 2005).

3.1.1. Is the Expectations Formation Realistic?

As described in Brayton et al. (1997), the most recent Federal Reserve model, the FRB/WORLD, also employs non-fully rational expectations.

Recognizing the uncertainty surrounding expectations formation, the main Fed model not only uses model consistent rational expectations, but it also models expectations as derived from a VAR for five 'core' macro variables (federal funds rate, CPI inflation, output gap, long-run inflation expectations and long-run interest rate expectations). The underlying justification is that the agents understand the main features of the economy, as represented by small economic models, and use this information to form their expectations. The Fed model, however, does not currently incorporate time-variation in the parameters.

The realism of forming expectations by adaptive learning can be gauged by looking at survey-based expectations. The expectations derived according to the adaptive learning algorithm track pretty well the inflation expectations from the Survey of Professional Forecasters. In particular, they replicate the underestimation of inflation in the two peaks in the 1970s and the overestimation of inflation during most of the 1990s. Less successful in tracking survey-based forecasts are purely adaptive (naïve) expectations and RE (since the data display large and persistent forecast errors, which are less consistent with RE). A substantial literature, in fact, emphasizes how survey expectations reflect an intermediate degree of rationality, rejecting full rationality as well completely naïveté (see Roberts 1998 for example).

Adam (2005) provides some experimental evidence on the formation of inflation expectations, showing that forecast rules in which agents condition on lagged inflation successfully mimic the inflation expectations of the subjects in his experiment.

Moreover, learning with a constant gain, useful to account for unknown structural breaks, seems a plausible choice to model the behavior of professional forecasters. Branch and Evans (2005) show that constant gain models of learning fit forecasts from surveys better than other methods for both inflation and output growth. They find that constant-gain learning models dominate models with optimal constant gain (obtained by minimizing the forecasts' Mean Square Error), with Kalman Filter, and with Recursive Least Squares learning. Their results therefore support constant-gain learning as a model of actual expectations formation.

3.2. Real Marginal Cost as the Driving Variable

Following recent research (Gali' and Gertler (1999), Sbordone (2003)), I also experiment the real marginal cost as the relevant driving variable for inflation

$$\pi_t - \gamma\pi_{t-1} = \frac{(1-\beta\alpha)(1-\alpha)}{\alpha} s_t + \beta E_t\left[\pi_{t+1} - \gamma\pi_t\right] + u_t \tag{22}$$

which can be rewritten as

$$\pi_t = \frac{\gamma}{1+\beta\gamma}\pi_{t-1} + \frac{\beta}{1+\beta\gamma}\widehat{E}_t\pi_{t+1} + \frac{(1-\beta\alpha)(1-\alpha)}{\alpha\left(1+\beta\gamma\right)} s_t + u_t \tag{23}$$

As in the previous case, I assume the same PLM for firms, i.e. firms form their expectations by estimating an AR(1) specification for inflation as in (16).

The ALM for inflation becomes

$$\pi_t = \frac{\beta\phi_{0,t}\left(1+\phi_{1,t}\right)}{1+\beta\gamma} + \frac{\gamma+\beta\phi_{1,t}^2}{1+\beta\gamma}\pi_{t-1} + \frac{(1-\beta\alpha)(1-\alpha)}{\alpha\left(1+\beta\gamma\right)} s_t + u_t \tag{24}$$

4. SOME SIMPLE IRRATIONAL EXPECTATIONS ECONOMETRICS: EMPIRICAL RESULTS

I can now estimate the inflation equation, assuming that firms form expectations and update their beliefs through constant-gain learning as described. I use quarterly U.S. data on inflation, output, and real marginal costs from 1960:01 to 2003:04. Inflation is defined as the annualized quarterly rate of change of the GDP Implicit Price Deflator, the output gap as detrended GDP after removing a quadratic trend, the real marginal cost as the unit labor cost, which is empirically proxied by the log labor income share in deviation from the steady state. All data are taken from FRED, the database of the Federal Reserve Bank of Saint Louis. I allow agents to initialize their estimates of coefficients, variances, and covariances, using pre-sample data from 1951 to 1959.

The empirical exercise proceeds as follows. First, I estimate the agents' PLM allowing them to learn the forecasting coefficients over time through constant-gain learning, as neatly described by expressions (18) and (19). Then, I substitute the resulting forecasts into the original inflation equations, (16) and (22). I initially fix the constant gain κ at the value of 0.015. This is consistent with values derived by minimizing the deviation of the constructed series from the expected inflation series from the Survey of Professional Forecasters, as found by Orphanides and Williams (2004).

I can then simply estimate the ALM for inflation by NLLS (Nonlinear Least Squares). In this way, I am able to *disentangle* the effects of learning from the effects due to structural sources of persistence in inflation, such as price indexation. Notice that having explicitly modeled the formation of agents expectations, it is not necessary to estimate the inflation equation by GMM as under RE, therefore avoiding the criticism that GMM has received for Phillips curve estimation (Lindé 2002, Jondeau and Le Bihan 2003, Fuhrer and Olivei 2004).

This paper therefore focuses on a single equation estimation of inflation dynamics. This approach avoids infecting the inflation equation by potential misspecifications in other parts of the model. In a companion paper (Milani (2004b)) I jointly estimate, instead, a full New-Keynesian model with learning by likelihood-based Bayesian methods. The two papers represent different thought experiments: this paper is implicitly assuming that agents' learning is correctly modeled and it focuses on estimating structural parameters *given* the assumed learning specification. Milani (2004b) pushes the experiment a step further and aims to *jointly* extrapolate from the data the learning rule coefficients together with the structural coefficients. In that way, agents' beliefs and their learning speed are jointly estimated together with the rest of the model. Structural estimates will be affected by the uncertainty concerning agents' learning specification. This allows for a better account of total uncertainty in the system. The drawback is that misspecifications in the learning equation will bias the rest of the model coefficients. The current paper instead separately estimates the learning equation; when the results are inserted in the ALM, they are treated as certain. Therefore, the standard errors of the structural coefficients are likely to underestimate the true underlying uncertainty.

These papers provide a first example of what Ireland (2003) has defined "Irrational Expectations Econometrics", judging it needed to complement the mainly theoretical results of the previous adaptive learning literature.

4.1. What Creates Persistence in Inflation?

The inflation equation is quite general, allowing for both indexation by firms and non-rational expectations. Whether the persistence in inflation is structural or due to learning behavior becomes then an empirical question. Understanding the sources of persistence is crucial and the results can affect the recommendations for optimal monetary policy.

4.1.1. Agents' Beliefs

Figure 1 shows the evolution of the estimated coefficients in the agents' forecasting equation (agents' PLM). Economic agents are updating the coefficients through CGL. The reported evolution of beliefs is obtained assuming a constant gain equal to 0.015.

In the beginning of the sample, agents were coming from periods of low and volatile inflation (the 1950s and 1960s), and, consequently, they estimated low autoregressive coefficients for inflation (around 0.15 at the beginning of the sample). In the 1970s, inflation rose substantially and also became more persistent. Agents recognized the shift and in the 1970s they start estimating much larger autoregressive coefficients (with a pick around 1975, where the estimated $\phi_{1,t}$ went up to 0.958). The estimates of perceived inflation persistence declined in the last part of the sample, though remaining above 0.8.

The evolution of the intercept in the inflation equation (recalling that the true value should always be 0, being the variables demeaned) indicates that the perceived inflation target was low in the 1960s, increased and remained high through the second half of the 1970s, and constantly decreased after the Volcker's disinflation.

4.1.2. Structural Estimates

Table 1 shows the estimation results. The table reports the estimates for the alternative specifications with the output gap and the real marginal cost as main driving variables for the dynamics of inflation, and for different values of the constant-gain coefficient ($\kappa = 0.015$, 0.02, and 0.03).

I obtain coefficients on indexation to lagged inflation, γ, equal to 0.139 and 0.047 in the output gap and real marginal cost equations, for the case with $\kappa = 0.015$. The estimates are small and not significantly different from 0. I estimate δ, the sensitivity of inflation to changes in the output gap, equal to 0.22. In the equation with real marginal costs as the driving variable, it is possible to obtain an estimate of α, the Calvo parameter. I estimate $\alpha = 0.671$, indicating prices that remain fixed for 3.04 quarters. With other gain coefficients, the results still indicate that inflation indexation is not supported by the data (I obtain γ equal to -0.001 and 0.045 when $\kappa = 0.02$, and γ equal to -0.18 and -0.09 when $\kappa = 0.03$). The results about indexation contrast with the estimates typically computed in the literature.

Likewise, I can estimate a reduced form equation for inflation, given by

$$\pi_t = \omega_b \pi_{t-1} + \omega_f \widehat{E}_t \pi_{t+1} + \delta y_t + \varepsilon_t \tag{25}$$

where $y_t = x_t, s_t$. This is similar to reduced forms usually estimated in the empirical literature (the New Hybrid Phillips curve for example) and the only difference comes from the use of subjective expectations instead of RE. Estimation of this simple equation yields

$$\pi_t = \underset{(0.15)}{0.105} \pi_{t-1} + \underset{(0.19)}{0.992} \widehat{E}_t \pi_{t+1} + \underset{(0.037)}{0.201} x_t + \widehat{\varepsilon}_t \tag{26}$$

or

$$\pi_t = \underset{(0.166)}{0.045} \pi_{t-1} + \underset{(0.197)}{0.972} \widehat{E}_t \pi_{t+1} + \underset{(0.029)}{0.147} s_t + \widehat{\varepsilon}_t \tag{27}$$

when the real marginal cost is used.

4.1.3. Comparison with the Literature

Let's compare γ, the estimated degree of indexation to past inflation, with other estimates in the literature. Christiano, Eichenbaum and Evans (2005) do not actually estimate γ, but they fix it to 1, indicating full indexation. Boivin and Giannoni (2003) and Giannoni and Woodford (2003), working with the same model of this paper, but with rational expectations instead of learning (in a fully specified model), estimate a coefficient of γ equal to 1. These results point towards extremely high levels of structural persistence in inflation.[7].

Therefore, estimates of γ close to 1 hinge somewhat on the assumption of rational expectations. When this assumption is weakened, by introducing small deviations from full rationality and allowing agents to learn over time, the degree of inflation persistence due to structural features of the economy can drop to almost zero.

There has been a considerable debate in the literature on whether inflation is mainly a backward or forward-looking phenomenon. Gali' and Gertler (1999) stress the importance of forward-looking expectations in their New Hybrid Phillips Curve (NHPC), although still obtaining a positive weight on lagged inflation. Fuhrer and Moore (1995) and Fuhrer (1997), on the other hand, depicts inflation as substantially backward-looking.

The results of this paper give merit to both ideas. In fact, I obtain that inflation is mostly forward-looking (and indeed very forward-looking as seen in eq. (26) and (27)). If expectations are formed as in this paper, however, the reduced form will be equivalent to a completely backward-looking specification. The effort to explicitly model subjective expectations gives a way to disentangle the persistence due to structural characteristics from those due to the sluggishness of forward-looking expectations (this is extremely hard with RE, or even impossible, see Beyer and Farmer 2004). It seems easy to understand, though, why many contrasting results in the literature have emerged disputing the relative importance of backward and forward-looking terms.

5. LEARNING SPEED AND INFLATION PERSISTENCE

In the estimation, I have experimented different gain coefficients between 0.015 and 0.03. Such values are common in empirical studies adopting constant-gain learning, as Orphanides and Williams (2003) for example. The degree of persistence introduced in the system by learning, as well as the estimates of the indexation parameter, are likely to be strongly dependent on the choice of the gain parameter.

For this reason, it becomes essential to investigate how the estimates of structural persistence vary across a wide range of possible gain values. This experiment allows me to examine the relationship between learning speed and inflation persistence.

Figure 2 shows such a relationship. The figure illustrates how the reduced form coefficient on lagged inflation ($\frac{\gamma}{(1+\gamma\beta)}$) varies with different gain values ranging from 0 (which is the limit case corresponding to RE) to 0.30 (the results for values above 0.30 are totally similar to those on that upper bound).

There seems to be a sort of V-shaped relationship. With a zero gain (remembering that $\kappa \to 0$ corresponds to RE), or with very small gains, the weight given to lagged inflation in an equation like (26) is sizeable. With slightly larger gains, the implied coefficients on the backward-looking term becomes much smaller and implies coefficients below 0.2 for gains around 0.025. Inside this range, learning is successful in creating enough persistence in inflation, so that no role remains for additional sources of structural persistence. Outside that range, for lower or larger gain values, backward-looking components and indexation remain important, since learning with those gains does not generate expectations of future inflation rates that seem supported by the data.

6. LEARNING SPEED AND FIT

Having such a diverse range of results, it is important to evaluate which value of the constant gain is more supported by the data. This information is attained by estimating the following simple equation

$$\pi_t = \delta y_t + \beta \widehat{E}_t \pi_{t+1} + u_t, \text{ where } y_t = x_t, s_t \tag{28}$$

over the range of all possible gain coefficients from 0 to 0.30 and evaluating how the in-sample fit changes. It is assumed again that agents form expectations estimating autoregressive specifications for inflation and updating the coefficients over time through constant-gain learning. As a measure of fit, I report the Schwartz's Bayesian information Criterion (BIC).

Figure 3 indicates how the fit (BIC) varies across the whole range of assumed gain coefficients. Again, it is possible to observe a sort of V-shaped relationship. The best-fitting specifications (lowest BIC) have gain coefficients in the range typically used in previous calibrated learning studies (between 0.015 and 0.03). Very small gains (close to 0) and large gains (above 0.06) perform very poorly in terms of in-sample fit.

Table 2 reports the gain coefficients in correspondence of which the lowest BIC is derived. The best-fitting constant gain coefficient equals 0.02 for the inflation equation with the output gap as driving variable. Such a value is similar to what found by Orphanides and Williams (2004) minimizing deviations of their model-based expectations from data on survey-based expectations. The results are similar if real marginal costs replace the output gap as the driving variable for inflation. The best-fitting constant gain coefficient, reported in table 2, now equals 0.025. Figure 7 also illustrates a similar relationship between constant gain and fit.

6.1. Learning Speed: Pre-1979 and Post-1982 Samples

So far, I have derived the value of the constant gain coefficient that gives the best in-sample performance for the whole 1960-2004 estimated sample. It is interesting, however, to split the sample and assess if the speed at which agents were learning have been substantially different over time. The pre-1979 sub-sample has been described as a period characterized by a passive and destabilizing monetary policy rule by Clarida, Gali, and Gertler (2000), who found instead an active and stabilizing monetary policy rule after 1982. Other papers support the existence of two different policy regimes in the pre-1979 and post-1982 periods. The 1960s and 1970s have also been characterized by high-volatillity shocks, whereas the 1980s and 1990s have been hit by more modest shocks: researchers refer to this decline in macroeconomic volatility as "the great moderation". Being the economic environments different in the two sub-samples, it is also possible that agents' learning speeds have been different.

Figure 4 and 5 show the fit of the inflation equation (28) across the whole range of values of the constant gain coefficient for the pre-1979 and post-1982 samples. The best-fitting gains are also reported in table 2. The results for the pre-1979 sample are similar to those regarding the full sample. The best-fitting gain coefficient in the pre-1979 sample is estimated at 0.019. Figure 5 instead highlights how much larger gains are preferred by the data in the Volcker-Greenspan period. The best-fitting gain in the post-1982 sample is, in fact, equal to 0.122.[8] Learning seems to have been much faster in the 1980s and 1990s compared to the previous decades. To facilitate intuition, the gain can be interpreted as an indication of the number of observations agents use to form their expectations. A gain equal to 0.019 indicates that agents use roughly 13 years of data (52. 6 quarters), whereas a gain equal to 0.122 indicates that agents use 2 years (8.2 quarters) of data to form their expectations. Similar results are obtained estimating the inflation equation with real marginal costs replacing the output gap. The best-fitting constant gains assume value 0.0245 in the pre-1979 sample and 0.124 in the post-1982 sample. Figure 8 and 9 also depict a similar relation between learning speed and fit across the two sub-samples.

This section has provided some evidence that the speed of learning of agents has importantly changed over time. It would be interesting to study how different learning speeds depend on or affect monetary policy or the level of macroeconomic volatility. Those would be useful extensions, but beyond the scope of the paper.

7. ROBUSTNESS

I have so far assumed that economic agents use a simple autoregressive model as in (17) as their PLM. But the correct model of expectations formation is uncertain. Therefore, in this section I aim to evaluate the robustness of my results to alternative forecast rules.

7.1. Phillips Curve as Learning Rule

Suppose now that private agents use the reduced form of a Phillips curve to form their inflation expectations (assuming they have information up to $t-1$). Again, they do not know the relevant parameters, so they gradually learn them from the data they observe. The agents' PLM is now

$$\pi_t = \phi_{0,t} + \phi_{1,t}\pi_{t-1} + \phi_{2,t}x_{t-1} + \varepsilon_t \tag{29}$$

with the output gap, and

$$\pi_t = \phi_{0,t} + \phi_{1,t}\pi_{t-1} + \phi_{3,t}s_{t-1} + \varepsilon_t \tag{30}$$

with real marginal costs entering the Phillips curve. Now the agents are using the same variables that enter the actual law of motion of inflation in the model.

Under a constant gain equal to 0.02 (the best-fitting gain in the previous case), for example, agents estimate the Phillips curve coefficients reported in figure 10. The intercept again starts low and increases over the sample, decreasing only in the second half of the 1990s. The agents' belief about the persistence of inflation is low in the 1960s and early 1970s, when it jumps from below 0.4 to almost 1 (0.96) around 1975. Another period of extremely high persistence occurs in 1981-1982. Therefore private agents perceive an extremely large degree of persistence both during the run-up of inflation in the 1970s and during Volcker's disinflation in the early 1980s. The third coefficient in the graph represents the elasticity of inflation to the output gap (slope of the Phillips curve). This elasticity has been increasing from the late 1960s until 1975, it remained stable (around 0.2) in the second part of the 1970s, and then it declined after Volcker's disinflation. In the 1980s and 1990s, the estimated elasticity has always been low and it has fallen even more after 1998 (stabilizing around 0.06). The dynamics is consistent with the existence of an important trade-off between inflation and output in the 1960s-1970s, which almost disappeared in the 1980s-1990s.

If I estimate the degree of indexation under this alternative learning rule, I obtain similar results. Figure 11 shows the estimated backward-lookingness in inflation across gain coefficients. The estimates are close to 0 for a range of gain values above 0.05. The figure also superimposes the fit of the different gains. The best-fitting gains are now larger than those obtained under the simpler AR(1) forecasting rule. The best-fitting gain, reported also in table 3, equals 0.068 (in correspondence of this gain, the coefficient on the backward-looking term in the inflation equation equals 0.043).

Figure 12 instead repeats the exercise for the learning rule in which marginal costs replace the output gap. Apart from the weird shape, the message is similar. The best-fitting gain is now 0.0355 (larger than before) and the estimated persistence coming from structural features is small (but larger than before, 0.168). It is worth noticing from figures 11-12 that the models with the output gap seem to provide a better fit than do those with real marginal costs (BIC is around 2.6 instead of 3).

7.2. *Interest Rates in the Learning Rule*

Let's now suppose agents use even more information in their forecasting rule. I now assume they also include nominal interest rates (the federal funds rate) to forecast future inflation. The PLM is

$$\pi_t = \phi_{0,t} + \phi_{1,t}\pi_{t-1} + \phi_{2,t}x_{t-1} + \phi_{3,t}i_{t-1} + \varepsilon_t \tag{31}$$

and it is similar to inflation equations that commonly make part of estimated monetary VARs, for example.

The results reported in figure 13 show that the best-fitting gains (also corresponding to the lowest levels of indexation) lie around 0.1. The best-fitting gain is estimated equal to 0.0995 (at this gain the backward-looking term in inflation equals 0.23). Although not a general rule, from the cases examined so far, it appears that larger information sets lead to higher estimated speeds. The constant gain increases in fact from 0.02 when only lagged inflation enters the learning rule, to 0.068 when inflation and output gap enter, to 0.0995 when inflation, output gap, and the federal funds rate enter.

The results suggest that estimates of the gain are likely to depend on the assumed learning rule. But also the choice of the regressors entering the agents' PLM can be based on the fit of the various choices. And it is worth noticing that the learning specifications all fit better than an entirely backward-looking inflation equation, where current inflation is regressed on lagged inflation and current output gap (BIC = 3.18). In all cases, learning seems to provide a way to account for the persistence of observed inflation. Future research should shed more light on how to best model the learning process.

8. POLICY IMPLICATIONS

The scope of the paper has been so far descriptive. But understanding the main sources of inflation persistence is also crucial from a normative standpoint. Whether the inertia in inflation is structural or is instead due to the way agents form their expectations affects in fact the optimal monetary policy. If the mechanisms that are introduced in the models to induce persistence, such as indexation, turn out to be the wrong representation of the economy, then a welfare analysis based on such microfoundations will be erroneous as well.

8.1. Optimal Monetary Policy with Structural Inflation Persistence

Suppose first that inflation depends on its lagged values because of automatic indexation by firms. Suppose expectations are fully rational. In this case, the rigidity of prices is due to structural characteristics of the economy.

The optimal monetary policy in the case of indexation, considering a microfounded DSGE model as in Woodford (2003b), could be implemented by a central bank minimizing a welfare-based loss function which takes the following form

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} L_t \tag{32}$$

$$L_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda_x (x_t - x^*)^2 \tag{33}$$

where $\lambda_x = \frac{\delta}{\theta}$ under this paper's microfoundations, and $x^* > 0$ is the optimal output gap level, which depends on microeconomic distortions such as the degree of market power and the size of tax distortions in the economy.

This loss function is derived from the microfoundations of the model. It is therefore optimal not just to minimize the deviation of inflation from target (assumed equal 0 here), but also the rate of change. The more persistent inflation is, the more aggressive the optimal reaction of monetary policy would be. Optimal policy under commitment from the timeless perspective will satisfy the target criterion

$$\pi_t - \gamma\pi_{t-1} = -\frac{\lambda_x}{\kappa}(x_t - x_{t-1}) \tag{34}$$

The optimal rule would be given

$$i_t^* = \psi_\pi \pi_{t-1} + \psi_x x_{t-1} + \psi_u u_t + r_t^n \tag{35}$$

where ψ_π, ψ_x, ψ_u are optimal feedback coefficients, i_t^* is the policy instrument, and r_t^n is the natural real rate of interest. Monetary policy therefore responds to past observable variables and current shocks.

8.2. *Optimal Monetary Policy with Adaptive Learning*

On the other hand, suppose that the persistence in inflation is not caused by structural features (assuming $\gamma \approx 0$), but it is instead due to learning by economic agents. In this case, if the central bank recognizes this, it would not be optimal to react so aggressively to inflation as if γ was close to 1. But it would become optimal, instead, to react to the private sector expectations of inflation. This would avoid fluctuations induced by mistaken expectations, not in line with the inflation target. The central bank would in fact want to avoid fluctuations that become unmoored from policy objectives. Let's consider the following New-Keynesian model with learning, also described in Evans and Honkapohja (2003), Preston (2003), and Milani (2004b)

$$\pi_t = \delta x_t + \beta \widehat{E}_t \pi_{t+1} + u_t \tag{36}$$

$$x_t = \widehat{E}_t x_{t+1} - \sigma \left(i_t - \widehat{E}_t \pi_{t+1} - r_t^n \right) \tag{37}$$

where I have added an aggregate demand equation, and where σ represents the elasticity of intertemporal substitution. The central bank will now target the more familiar welfare-based loss function

$$L_t = \pi_t^2 + \lambda_x \left(x_t - x^* \right)^2 \tag{38}$$

leading to an optimal target criterion similar to (34), but with $\gamma = 0$.

The optimal policy rule under commitment in this case would be (as in Evans and Honkapohja 2003)

$$i_t^* = \frac{1}{\sigma} \left[\widehat{E}_t x_{t+1} - \frac{\lambda_x}{\lambda_x + \delta^2} x_{t-1} + \left(\frac{\beta\delta}{\lambda_x + \delta^2} + \sigma \right) \widehat{E}_t \pi_{t+1} + \frac{\delta}{\lambda_x + \delta^2} u_t + \sigma r_t^n \right] \tag{39}$$

The implied reaction function makes clear the need for the central bank to respond now to expectations of the relevant variables. Expectations are taken as given by the policymaker, who does not incorporate the agents' learning rule in its optimization problem. Hence, I abstract here from issues of active experimentation. In this way, the optimal target criterion is satisfied regardless of the particular expectations held by private agents.

In such a framework, a fundamental task of the central bank and optimal monetary policy becomes *the management of expectations*, as emphasized by Woodford (2003b).

In order to keep inflation expectations close to target, the importance of *transparency* and *credibility* should be emphasized. In particular, if the monetary authority lacks credibility, every attempt to reduce inflation, not believed by the public (not incorporated in $\widehat{E}_t\pi_{t+1}$), may be useless and inflation may just continue to rise for a long time. And a more transparent central bank is likely to facilitate private sector learning. Bad policy therefore in this new framework can mean unsuccessful management of expectations and can arise also in the case of a central bank following a truly optimal rule derived from dynamic optimization. The undesirable outcome of policy in such a case would be due to an important misspecification of the policymaker's model: the failure to understand and incorporate the way agents form their expectations.

Since an important concern in monetary policy points towards robustness of a chosen policy rule, it would also be necessary to examine the effects of optimal rules under learning, if the policymaker does not recognize the true agents' PLM or assumes that they form expectations rationally. Preston (2003) shows that price-level targeting is more robust than inflation targeting if the policymaker wrongly assumes that agents have RE rather than recognizing their learning rule. Similarly, it would be important to evaluate the losses of optimal policy if the policymaker thinks persistence as structural whereas it is due to learning and vice versa. Coenen (2003) shows the potential large losses from underestimating the degree of inflation persistence and suggests aggressive policy as a safety net. Certainly, if we are willing to believe that bounded-rational expectations and learning behavior are important determinants of inflation dynamics, new avenues of research about optimal monetary policy open. The robustness of optimal rules not only to standard model uncertainty, but also to uncertainty in the correct specification of the agents' learning rule (dropping RE opens a wide range of different alternatives), would be an important matter. Indeed, in real world policy-making, central banks would hardly argue to target loss functions as (33), which include the rate of change, besides the level, of inflation; policymakers put a lot of effort instead on monitoring the evolution of private sector expectations. This is more consistent with models where learning is important than with models with structural persistence.

9. CONCLUSIONS

Several papers have studied the determinants of inflation dynamics. Monetary policy models often include a law of motion for inflation in which current inflation depends on future expected inflation and current output gap or marginal costs. Such a relationship, to which researchers refer as New Keynesian Phillips curve, can be derived from the optimizing behavior of firms in models with imperfect competition and sticky prices. Firms, in those models, have rational expectations. In their simplest form, New Keynesian Phillips curves fail to match the persistence characterizing actual data on inflation. Therefore, researchers have proposed various extensions that lead to persistence in the inflation equation. Rule-of-thumb behavior, price indexation, menu costs are popular modeling devices to account for inflation inertia.

This paper has suggested a different approach, proposing learning as an important determinant of inflation behavior. In the model, economic agents form expectations from simple economic models, not knowing the true model parameters. They have the same knowledge that econometricians would have, and, therefore, they use historical data to infer the relevant parameters and update their estimates through constant-gain learning.

The paper shows that when learning replaces the standard assumption of rational expectations, structural sources of persistence, such as indexation, are no longer essential to fit the data. Therefore, learning seems to be a major source of persistence in inflation. Disentangling the role of learning from that of structural sources of persistence carries also normative implications. If learning rather than mechanical indexation is the main source of persistence in inflation, the implied optimal monetary policy will be different.

Under constant-gain learning, one's results can heavily depend on the choice of the gain. In the paper, I have shown how the estimated backward-lookingness in inflation varies over the range of possible gain coefficients. The large differences are evidence that working with estimated, rather than arbitrarily chosen, constant gain coefficients is necessary. The paper has provided some preliminary evidence, calculating the best-fitting constant gain and showing how the fit of the inflation equation changes across the range of assumed gains. The best-fitting gain for the full sample considered in the paper seems to be around 0.02, not far from values used by other studies in the learning literature. The estimated learning speed, however, has been far from constant: the best-fitting constant gain has increased from values around 0.02 in the pre-1979 sample to values around 0.12 in the post-1982 sample, indicating a substantial increase in the speed at which economic agents are learning. The causes and consequences of the variation in learning speed need to be studied in future work.

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	Structural Coefficients (Stand.Errors)					
	κ= 0.015		κ= 0.02		κ= 0.03	
	with output gap	with marg. cost	with output gap	with marg. cost	with output gap	with marg. cost
γ	0.139 (0.21)	0.047 (0.18)	-0.001 (0.15)	0.045 (0.17)	-0.183 (0.095)	-0.093 (0.12)
β	1.125 (0.06)	1.019 (0.05)	1.114 (0.04)	1.01 (0.05)	1.088 (0.026)	1.00 (0.035)
δ	0.229 (0.04)		0.222 (0.032)		0.199 (0.024)	
α		0.671 (0.02)		0.679 (0.02)		0.694 (0.018)

Table 1 - Estimates Model with Learning. Equations estimated by NLS.

	Output gap as driving variable			Marg. cost as driving variable		
	Full-sample	Pre-1979	Post-1982	Full-sample	Pre-1979	Post-1982
Constant-Gain κ	0.02	0.019	0.122	0.025	0.0245	0.124

Table 2 - Best fitting constant-gain coefficient κ. *Note:* the estimated equation is $\pi_t = \delta x_t + \beta \widehat{E}_{t-1}\pi_{t+1} + u_t$, where $\widehat{E}_{t-1}\pi_{t+1} = \phi_{0,t-1}\left(1+\phi_{1,t-1}\right) + \phi_{1,t-1}^2\pi_{t-1}$.

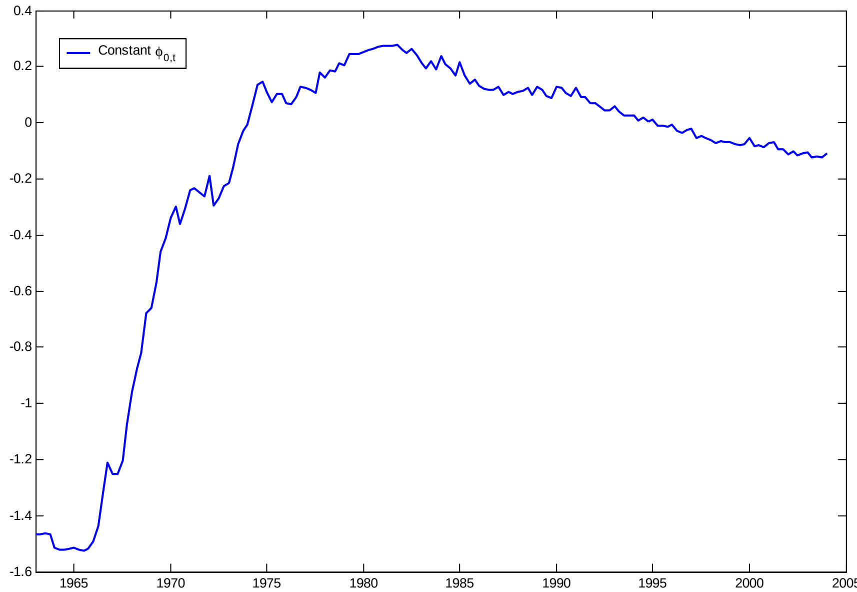
	Output gap as driving variable		Marg. cost as driving variable
	(1)	(3)	(2)
Constant-Gain κ	0.068	0.0995	0.0355
Indexation term $\gamma/(1+\gamma\beta)$	0.0426	0.23	0.1681

Table 3 - Best-fitting gain coefficients. *Note:* learning rules

(1) $\pi_t = \phi_{0,t}+\phi_{1,t}\pi_{t-1}+\phi_{2,t}x_{t-1}+\varepsilon_t$

(2) $\pi_t = \phi_{0,t}+\phi_{1,t}\pi_{t-1}+\phi_{2,t}s_{t-1}+\varepsilon_t$

(3) $\pi_t = \phi_{0,t}+\phi_{1,t}\pi_{t-1}+\phi_{2,t}x_{t-1}+\phi_{3,t}i_{t-1}+\varepsilon_t$



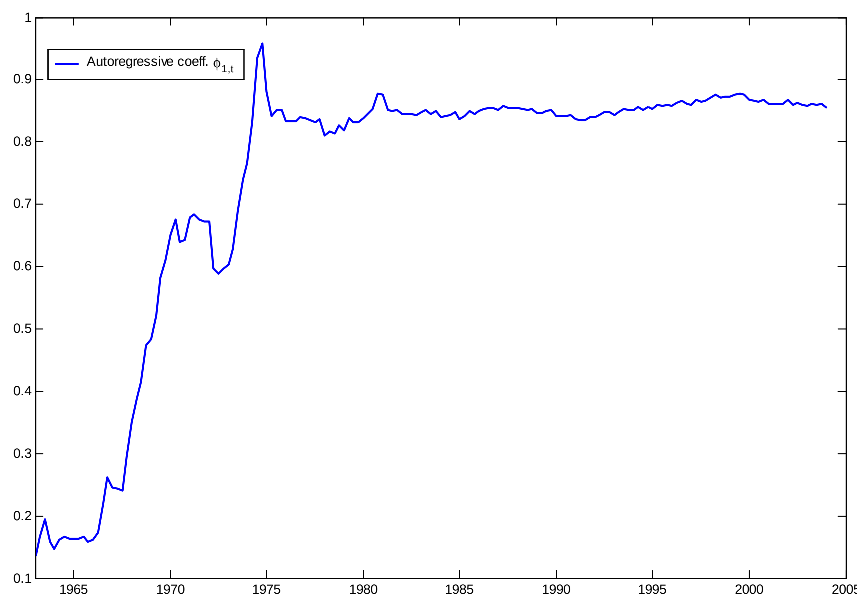


Figure 1 - Evolution of agents' beliefs over time.

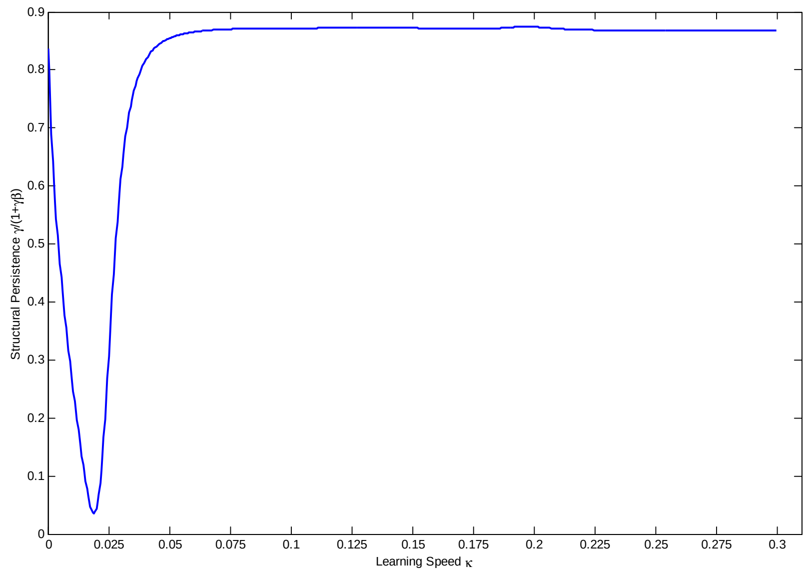


Figure 2 - Estimate of structural persistence parameter across constant gain coefficient values κ (inflation equation with output gap).

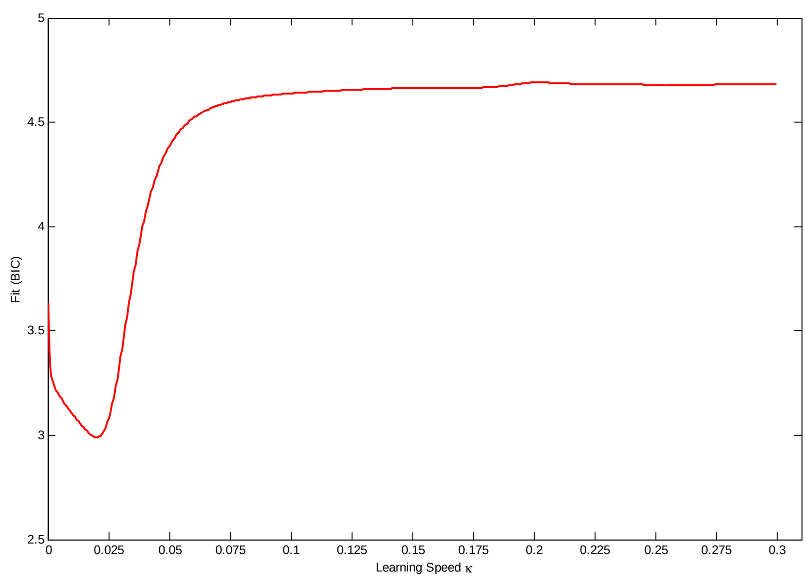


Figure 3 - Fit across constant gain coefficient values κ (inflation equation with output gap). Fit is measured by BIC.

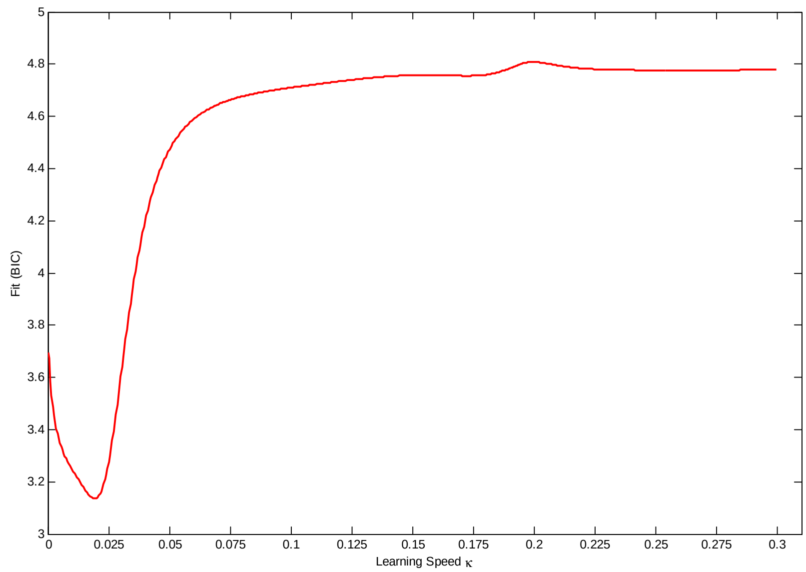


Figure 4 - Fit across constant gain coefficient values κ: pre-1979 sample (inflation equation with output gap).

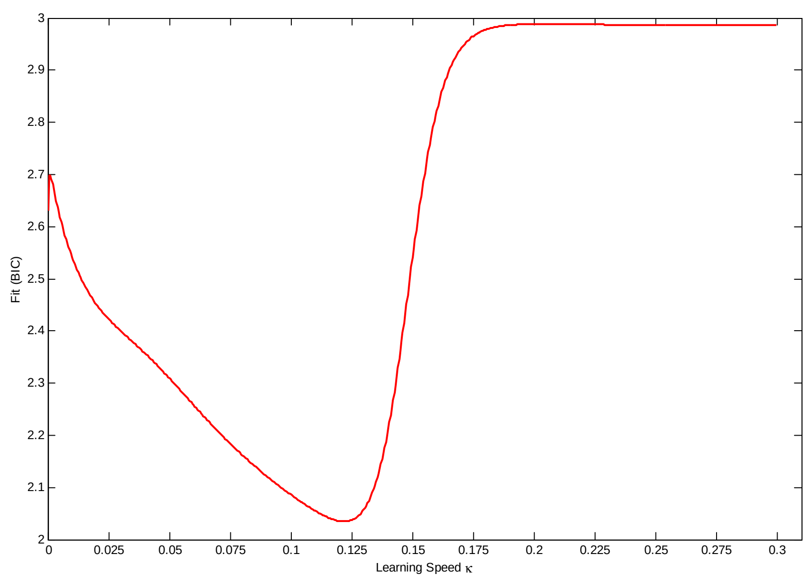


Figure 5 - Fit across constant gain coefficient values κ: post-1982 sample (inflation equation with output gap).

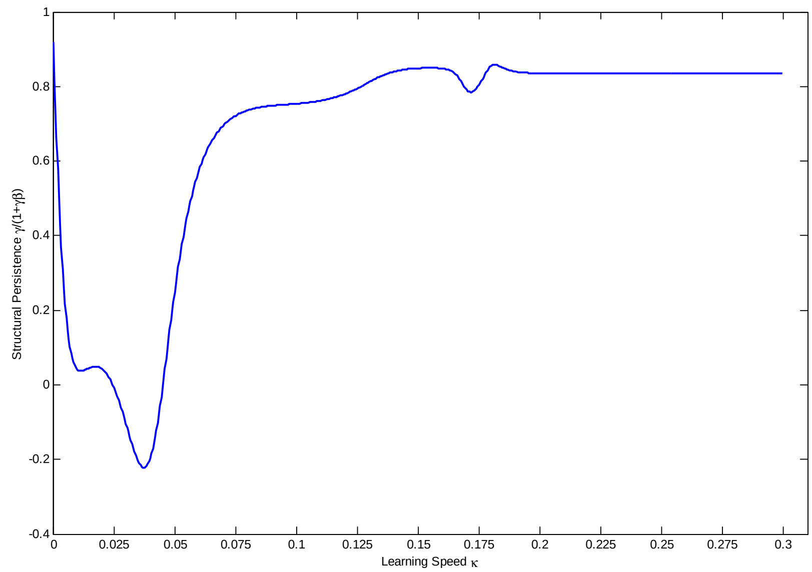


Figure 6 - Estimate of structural persistence parameter across constant gain coefficient values κ (inflation equation with marg. cost).

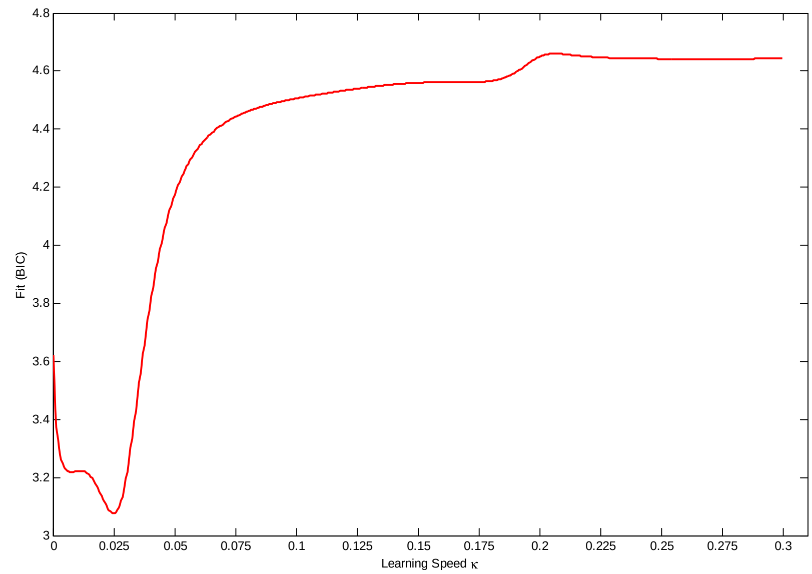


Figure 7 - Fit across constant gain coefficient values κ (inflation equation with marg. cost).

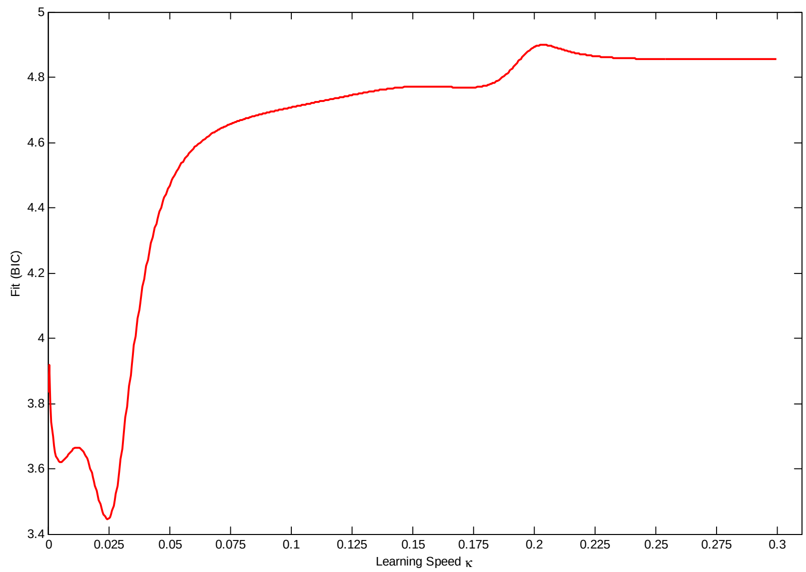


Figure 8 - Fit across constant gain coefficient values κ: pre-1979 sample (inflation equation with marg. cost).

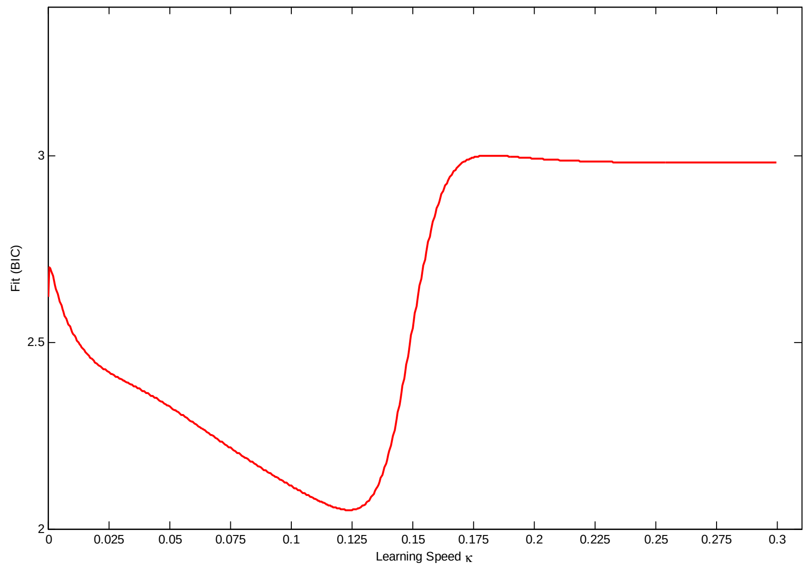


Figure 9 - Fit across constant gain coefficient values κ: post-1982 sample (inflation equation with marg. cost).

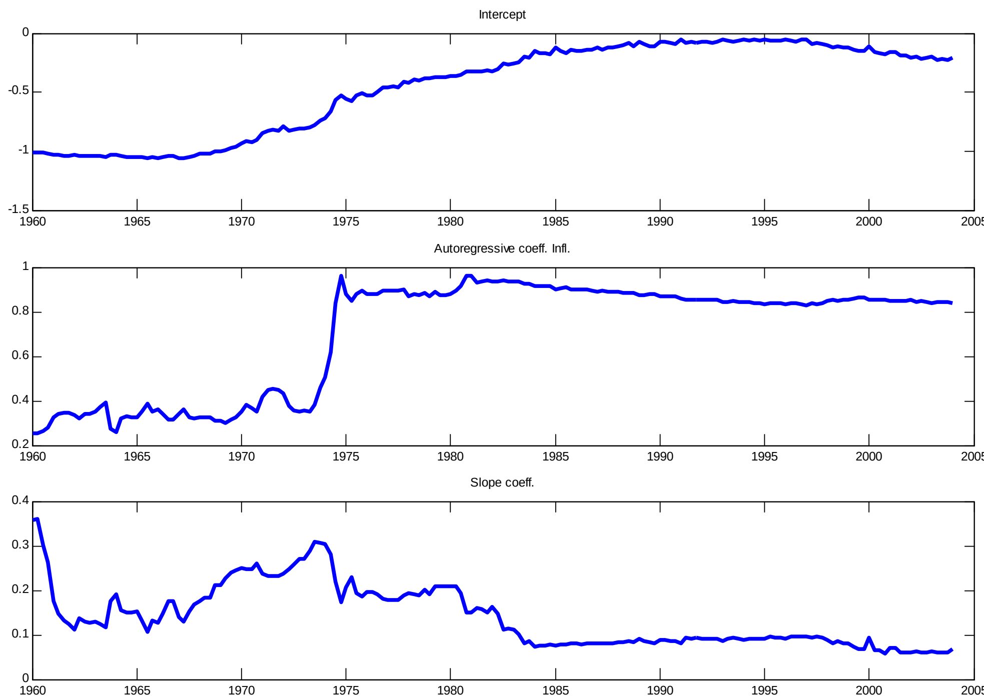


Figure 10 - Evolution of agents' beliefs (learning rule with output gap)

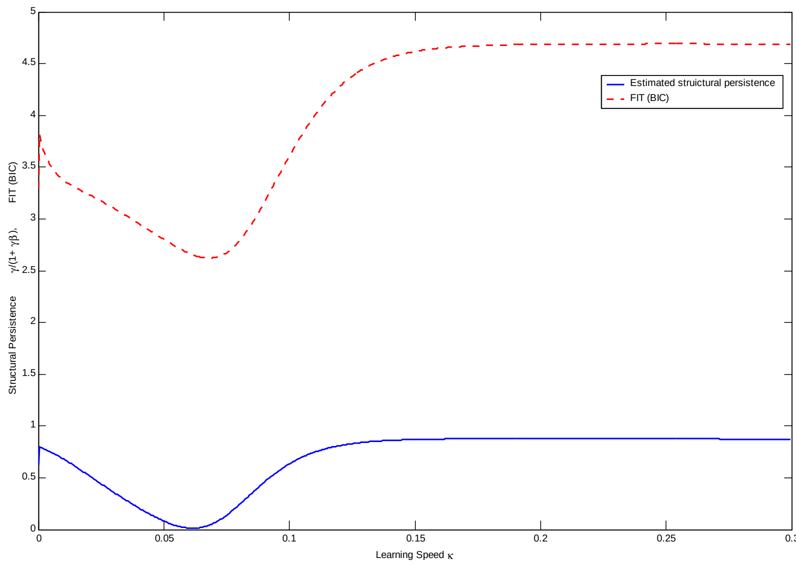


Figure 11 - Fit (dashed) and estimated persistence (solid) across constant gain coefficient values κ (inflation equation with output gap/ Phillips curve learning rule).

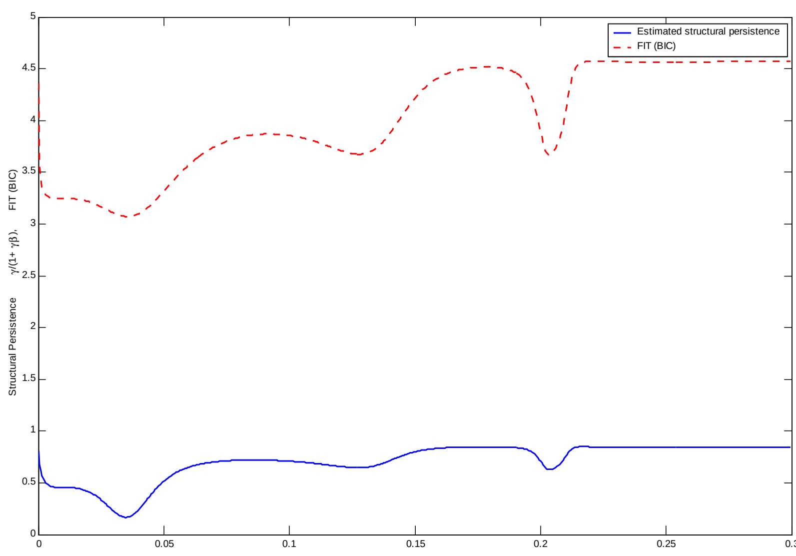


Figure 12 - Fit (dashed) and estimated persistence (solid) across constant gain coefficient values κ (inflation equation with marginal costs/ Phillips curve learning rule).

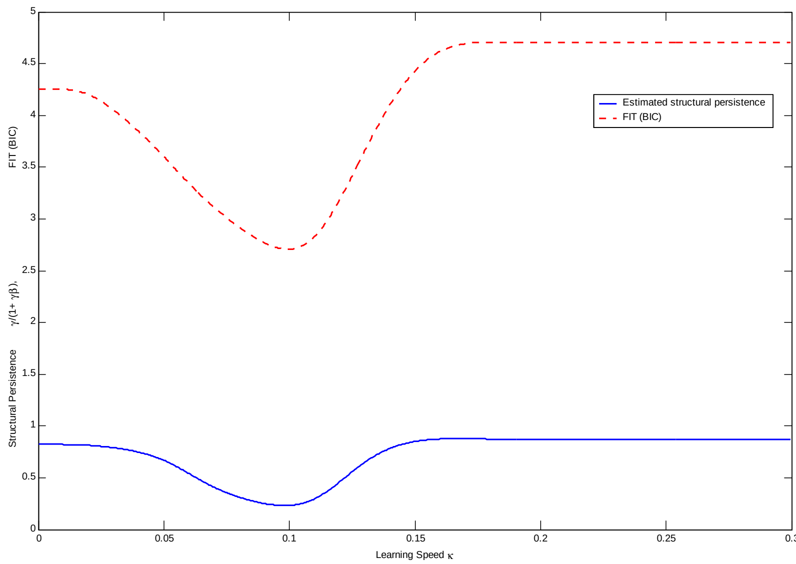


Figure 13 - Fit (dashed) and estimated persistence (solid) across constant gain coefficient values κ (learning rule with interest rates).

Notes

[1]Intrinsic in the sense that inflation persistence is a 'stylized fact' that we should expect in most industrialized economies.

[2]The full consumer maximization problem with learning is described in companion papers (Milani 2004a,b), which also examine time non-separable preferences.

[3]Consistently with most of the New Keynesian literature I log-linearize around a zero steady-state for inflation. I am therefore abstracting from the complications arising from log-linearizing around a positive inflation steady-state. For an account of the possible implications of this choice, see Kiley (2004) and Ascari (2004). Indexation permits to avoid the problems due to trend inflation, as shown in the appendix A of Ascari (2004). However, the present paper works with partial indexation, remaining partly vulnerable to such problems.

[4]I will omit '$\hat{}$' from the following section to save some notation. I also omit 'i' as a superscript to indicate the i-th firm, being the problem identical for every firm.

[5]For a different approach of considering learning, see Preston (2003), where learning is introduced directly from the primitive assumptions of multi-period decision problems. The derived law of motion for inflation will be equal to (10). My choice is instead similar to most papers in the adaptive learning literature (Evans and Honkapohja 2001, 2003, Bullard and Mitra 2002, Bullard and Eusepi 2003, and Williams 2003 are examples). See Milani (2004b) for estimations considering both approaches.

[6]According to an *anticipated utility model*, each period agents maximize their expected utility taking their beliefs and the model as constant, although the model is recursively estimated. But when more data become available, agents update their beliefs and use this new knowledge to maximize expected utility. Agents are therefore learning, but they are not involved in active experimentation as a fully rational behavior would imply.

[7]Notice that estimates of γ close to 1 imply reduced-form coefficients on the backward-looking term for inflation only around 0.5.

[8]This is obtained by using the estimated mean and variances of beliefs up to 1982:IV as initial conditions for the post-1982 learning algorithm. The experiment therefore refers to the case of agents that learn at the speed estimated in table 2 until the end of 1982 and then switch to the new speed constant gain afterwards.