Prices, Quantities, or Budgets: Optimal Policy Under Uncertainty

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Abstract

A government can use several mechanisms to induce firms to behave in the way it desires. It can set a subsidy rate for the desired activity, it can require that a set quantity of the activity be engaged in, or it can set a budget, adjusting either the subsidy rate or the quantity level to meet the budget. We demonstrate that uncertainty can make a budget policy more efficient than either a fixed subsidy rate or a fixed quantity. We also show that the optimal budget declines with a mean-preserving spread in the distribution of marginal costs.

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1. Introduction

Much regulation aims to control externalities in the presence of uncertainty. A large and influential literature considers how price and quantity mechanisms, and hybrid combinations of the two, perform.[1] This paper analyses a third option, budget regulation, in which firms choose quantity in response to a price government sets so that total spending by government on the program (or total revenue it collects) meets a specified target. We shall see that such a policy can be more efficient than both regulation by price or regulation by quantity.

Budget constraints are used by firms to manage divisions, departments and even some individual contracts. For example, firms often impose a capital budget that requires managers to prioritize among a wide range of projects with apparently attractive rates of return. University faculty have found that, in a crunch, budgets rather than previously allocated positions dominate hiring opportunities. Universities and colleges commonly set a budget for financial aid, adjusting the grant amount to some students to meet the budget. Weitzman, in his seminal 1974 paper, questions the emphasis and preference by economists for price instruments, given the apparent ubiquity in industry of quantity instruments. The same query can be posed for budget instruments relative to both price and quantity.

One application of our analysis concerns regulation of greenhouse emissions. While a simple cap-and-trade system is the archetypal incentive-based quantity regulation, the language of the proposed legislation would instead implement a form of cap-and-trade that is effectively price regulation or budget regulation. For example, in the

[1] Weitzman (1974), Weitzman (1978) and Roberts and Spence (1976) established the basic structure of the models in this literature. The dynamic issues, and more complex modeling date from Pizer (1999) and Pizer (2002). A recent survey is Hepburn (2006).

United States, the Lieberman-Warner bill would establish a Carbon Market Efficiency Board to determine when the trading program of permits imposes "excessive net costs"[2] on the economy, in which case it could implement a series of cost relief measures, including expanding "the total quantity of emission allowances made available to all covered facilities at any given time..."[3] These provisions can be (loosely) interpreted to mean that the regulation would adjust so that the market value of permits meets a fixed value. Another proposed bill, the Waxman-Markey bill, contains procedures that allow a "reserve allowance fund" to be auctioned when permit prices reach a specified level. Depending on demand, either quantity or price regulation holds in the short run. [4]

All three policies we consider are found in practice, even in such limited areas as the promotion of renewable fuels. In the United States, the Renewable Portfolio Standards (RPS) program uses quantity regulation, requiring electricity distribution companies to procure 10–15% of their electricity from renewable facilities. A market exists for renewable energy credits that can satisfy all or part of RPS requirements. An example of regulation by price lies in a US program which allows investor-owned utilities to earn a tax credit for investing in generating plants that use renewable fuels. This is a so-called "tax expenditure" program, which is analytically equivalent to price regulation. A budget regulation program for the same purpose exists for public and rural electric cooperatives which build plants using renewable fuels: the Department of Energy receives an annual appropriation to subsidize these activities. The subsidy rate varies by

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[2] Lieberman-Warner, Section 2602 (b)(3)(iv).

[3] Lieberman-Warner, Section 2604 (a) (1) (F). The legislation specifies that the additional permits be "borrowed" from future years' allocations, so that over a fifty year period quantity is the ultimate constraint. The Waxman-Markey reserve allocation is also borrowed from future allocations.

[4] The hybrid quantity/price provisions as well as other permit-price reducing mechanisms can, if implemented properly, substantially reduce the cost of the program. See, e.g., Jacoby and Ellerman (2004), Newell, Pizer and Zhang (2005), Fell, MacKenzie and Pizer (2008), Fell and Morgenstern (2009), the National Commission on Energy Policy (2009).

year, depending on demand, so that total annual spending on the program remains within budget.[5]

This paper examines conditions under which budget regulation is more efficient than price and quantity regulation. We think that the analysis applies beyond regulation (for example, we might think of government policies to promote innovation, to encourage road construction, to promote health insurance, and so on.) We therefore shall speak of a budget policy rather than of budget regulation. But to be concrete, we shall illustrate with a discussion of a government inducing firms to abate emissions.

We find that a budget policy dominates a policy with a fixed subsidy rate or a fixed aggregate activity level if the uncertainty is mild, and if the marginal cost curve is neither too flat nor too steep compared to the marginal benefit curve. Under the simplified assumptions of our model, a budget policy is always more efficient than at least one of the other alternatives, if not both. In contrast to Weitzman's (1974) results on the relative efficiency of price and quantity policies, the extent of uncertainty is critical to determining when a budget policy is best. Furthermore, we show that, unlike the optimal subsidy or the optimal quantity under a price or quantity policy, the optimal budget for a budget policy declines when uncertainty increases.[6] Lastly, we establish conditions where a budget policy is both the efficient regulatory strategy and costs government less than the expected cost of an optimal tax-expenditure program.

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[5] see EIA (2008). Claims are prioritized into several categories. Reimbursement rates vary by category, and all are subject to pro rata diminution if claims exceed the budget. Of course, this system is only efficient if the categories separate projects according to marginal productivity. While there is arguably a correlation – the more novel projects receive a higher reimbursement rate – a case can be made that the categories are more commensurate with marginal political productivity than economic.

[6] Kolstad (1996) finds that the extent of uncertainty changes the optimality of price and quantity instruments in a dynamic setting with learning.

The paper proceeds as follows. The next section gives additional background about the budget instrument in the context of environmental and energy regulation. Section 3 presents the formal model and compares the three policies. Section 4 turns to optimal budget policy, and show how the size of the optimal budget varies with uncertainty. The paper concludes in section 5.

2. Background

A budget policy can work in several ways. In all cases, the government first sets a budget. Next, nature resolves the uncertainty over costs. Government can then follow a cap-and-trade approach, choosing a cap on emissions and either distributing the permits and establishing a market for trades or holding an auction.[7] The cap is chosen so that the cap times the permit price equals the budget. Alternatively, the government can set a subsidy such that the combined activity level for all covered entities results in claims that exhaust the budget.

In Weitzman's model and the subsequent related literature, uncertainty is modelled as a shock to marginal cost. The regulator must know both the expected marginal cost and expected marginal benefits curve to determine the optimal price or optimal quantity. The "incentive" feature of the policies allows cost minimization among heterogeneous polluters, but still requires the regulator to know something about the aggregate emissions technology. Budget regulation starts from the same premise, but requires in addition that government can adjust its subsidy rate to meet the budget constraint. One way for government to do this is to observe the random shock to costs

[7] At this point one might wonder why the agency doesn't set a cap to maximize welfare; the first-best solution. The assumption here, as with other agency studies, is that the incentives of the agency require prior action by the polity, as the agency prefers to spend its entire budget, whatever that may be.

contemporaneously with firms.[8] A plausible scenario that would conform to this requirement arises when the cost shock is largely due to macroeconomic conditions, as is argued by Philibert (2008; p. 18) for greenhouse gas emissions. Alternatively, government may observe spending on subsidies for a sample of firms or time, and use a tatonnement process to adjust the subsidy rate to conform to its budget constraint.

Price, quantity, and budget policies all require that the instrument be set in the presence of uncertainty and that flexibility is maintained for the other targets. For a price policy, both quantity and the government's tax receipts or spending under the policy– the budget – will depend on the resolution of uncertainty and cannot be fixed in advance. For example, the Car Allowance Rebate System (popularly known as "cash for clunkers"), a federal price policy program that offers consumers a fixed subsidy for buying a car with improved fuel efficiency, was, less than two weeks into the program in July 2009, some two billion dollars over its initial appropriation, at which time the program was saved by additional legislative action.[9] For a quantity policy, neither the price nor the budget is fixed. While much of the discussion surrounding climate regulation in the United States has focused on the potential for permit price spikes in a cap-and-trade system, the uncertainty is two-sided. Permit prices in the sulfur-dioxide cap-and-trade market during its first decade were far below projections.[10]

For budget regulation, we assume, consistent with both experience and political economy theory, that the agency will always spend its entire budget, whether or not the

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[8] More generally, the regulator needs to observe the component of the shock that pertains to firms at the margin.

[9] See, e.g., http://blogs.edmunds.com/strategies/2009/08/cash-for-clunkers-senate-approves-2-billion-in-extra-funding.html (accessed August 19, 2009).

[10] See Tatsutani and Pizer (2008).

action is consistent with economic efficiency *ex post.*[11] Thus, budgets here bind from both above and below, and neither the quantity nor the per-unit price will be known at the time that the budget is set.

We abstract from important concerns such as the cost to the government of implementing the different approaches *ex ante* and of enforcing or monitoring performance *ex post*.[12] In our discussion of a budget policy, we consider only spending by the government, ignoring the costs to firms of abating pollution.[13] Ours is a partial equilibrium analysis. Moreover, each of the instruments can be structured to impose costs on either (or both) the public or private sector, and this choice has important implications for efficiency which we do not consider here.[14] Within this context we can pose Weitzman's question: notwithstanding imperfections in all three instruments, which will yield the best expected efficiency outcome?

3. Modelling Price, Quantity and Budget Instruments

3.1 Subsidy policy and quantity policy

The exposition below is made simpler by considering a subsidy for abatement rather than a tax on emissions (where emissions can be reduced by abatement), though of

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[11] See, e.g., Niskanen (1974), Romer and Rosenthal (1979).

[12] An excellent discussion of management issues involved in the "tax expenditure" (price) versus "direct expenditure" (generally budget) is contained in Weisbach and Nussim (2004). Also omitted from our discussion are political feasibility and whether the regulatory instruments fruitfully align with cultural and altruistic incentives. An excellent overview and discussion of the importance and interrelationship of these issues for price and quantity regulation of the environment is contained in Hepburn (2006).

[13] The government may wish to ignore abatement costs to the firm because they are difficult to measure or to verify. A policy which depended on such unverifiable measures might lack credibility and could induce much litigation.

[14] Overviews of these and a host of other issues that are relevant for instrument choice in environmental regulation are contained in Cropper and Oates (1992), Goulder, Parry, Williams and Burtraw (1999) and Goulder and Parry (2008). Another limitation of our analysis is its emphasis on static efficiency. For dynamic efficiency, the impact of instruments on incentives to innovate are critical. Fischer and Newell (2008) analyze a broad array of environmental policy instruments from this perspective.

course one is equivalent to the other. So our statements about a subsidy policy can also be interpreted as a tax or a price policy.

Our analysis starts with the key result from Weitzman (1974): when marginal costs and marginal benefits are relatively unresponsive to changes in quantity, a subsidy policy is more efficient than a quantity policy.[15] Alternatively, when marginal costs and marginal benefits are sensitive to the quantity level, a quantity policy can dominate a subsidy policy. Intuitively, flat marginal benefits imply a constant benefit per unit, allowing a tax or subsidy to correct the externality. In contrast, steep marginal benefits imply a dangerous threshold that should be avoided—a threshold that is efficiently enforced by a quantity control.

The result is illustrated in Figure 1. Shocks to marginal costs are represented by the random variable θ; f(θ) is the symmetric distribution over θ with mean zero. We suppose that the marginal benefit curve intersects the expected marginal cost curve at the point (q, p) = (1, 1), and for each realization of θ, the marginal cost curve takes the form:[16]

$$MC(\theta) = \{(p, q)|\ p = q - \theta\}.$$

Line MC(0) depicts expected marginal cost, the horizontal line P_0 shows the optimal subsidy rate, and the vertical line Q_0 shows the optimal quantity. Point E lies at the intersection of P_0, Q_0, and MC(0), and is the expected outcome under both the subsidy

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[15] Weitzman and the subsequent authors who address this issue obtain results for the case where both marginal costs and marginal benefits are linear and homothetic in θ. These assumptions are employed in this paper as well.

[16] The slope restrictions can be relaxed. Let the expected marginal cost curve be any vector through the origin with positive slope and suppose the marginal benefit curve intersects the expected marginal cost curve at some point (p_o, q_o). By rescaling the coordinates so that $x = q/q_o$ and $y = p/p_o$ we obtain the identical structure and results as in section 4.

and quantity policy.[17] Line MC(θ) is an additional marginal cost curve for a non-zero realization of θ, point X is the outcome for a given θ under a quantity policy, and Z is the outcome under a subsidy policy.

Figure 1 shows a marginal benefit line MB* which has identical curvature to the marginal cost lines (or a slope of -1). Triangle EAX, the efficiency loss under the quantity policy, is congruent to triangle AZU, the loss under the fixed subsidy. Thus, a subsidy policy and a quantity policy generate identical efficiency losses. For marginal benefit lines that are steeper than MB*, efficiency losses under the quantity policy are smaller than under the subsidy policy; the opposite holds for marginal benefit curves that are flatter than MB*.

Because uncertainty enters as a shock to the marginal cost curve, changing the value of θ yields proportionate increases or decreases in the welfare loss triangles. Thus, the relative efficiency of the two policies is maintained for all non-zero values of θ, and, in consequence, for any distribution f(θ). This establishes Weitzman's general result that when marginal benefits exhibit relatively less curvature than marginal costs, regulation by price is more efficient than regulation by quantity, and vice versa.

3.2 Budget policy

Consider next the budget policy. As the expected cost to the government under the optimal subsidy policy is p_0q_0, we let this be the budget allocated to the bureau. The bureau can meet the budget constraint by choosing a per-unit subsidy so that profit-maximizing firms will respond to $\hat{p}$ with a quantity $\hat{q}$ such that the allocated budget is

[17] See Weitzman (1974). His formulation is more general than is presented here. We employ the simplification in order to model the more complex, non-linear budget policy.

exhausted. Or it can set a cap $\hat{q}$ on abatement such that the subsidy rate, $\hat{p}$, which induces such abatement results in the budgeted spending on subsidies:

$$\hat{p}\hat{q} = p_o q_o = 1$$

and:

$$\hat{p} = \hat{q} - \theta.$$

The budget constraint, shown in Figure 1, is the hyperbola B = {(q, p)| pq = 1}. For $\theta = 0$, subsidy, quantity, and budget policies yield the identical result. For other values of θ, the outcome under the budget policy lies at the intersection of the associated marginal cost curve and curve B (for example, point Y in Figure 1).

Because the budget constraint is non-linear, no single marginal benefit line will equate efficiency losses among the policies for all values of θ. However, there exists an indifference curve I_P such that when the optimal outcome lies on I_P (that is, MB and MC(θ) intersect at a point on I_P), efficiency losses are identical under subsidy and budget policies. If the optimal outcome would lie between I_P and P_0, subsidy policy is more efficient than budget policy; for first-best optimal outcomes between I_P and B, budget policy is more efficient. Curve I_Q is defined similarly.

I_P and I_Q are defined as follows: (the derivation is contained in the Appendix, Lemma 1 and Lemma 2):

$$I_Q = \{(q, p) \mid (q - ½)(q + p - 1) = ½\}$$

$$I_P = \{(q, p) \mid (p - ½)(q + p - 1) = ½\}$$

Both intersect the point E = (1,1). I_Q is asymptotic to the line p + q = 1 as θ becomes small, and approaches the line q = ½ for large values of θ. I_P is left-asymptotic to the line p + q = 1, and right asymptotic to the line p = ½. (See Figure 2.)

3.3 Comparing policies

Given the characterization of the curves I_P and I_Q, we can investigate the relative efficiencies of the three policies. We first consider the case where subsidy and quantity policies are equally efficient.

Proposition 1: Suppose the marginal benefit curve is such that expected social welfare is the same under the optimal subsidy rate and the optimal quantity. If the budget under the budget policy equals expected spending by government under the subsidy policy, then for every non-zero value of θ, the budget policy dominates both the subsidy policy and the quantity policy. Under these conditions, when the marginal cost curve is subject to any uncertainty, the budget policy dominates both the subsidy policy and the quantity policy.[18]

As Figure 2 shows, the optimal outcome for each θ, which lies on MB*, is always closer to B than to either P_O or Q_O. Compare first a budget policy to a subsidy policy. For negative shocks to the marginal cost of abatement (negative values of θ), MB* lies beneath B, whereas I_P is in between B and P_O. In these cases, both a subsidy policy and a budget policy result in too much abatement (excess q). But a budget policy moderates the excess in part by responding to low costs with a reduced subsidy rate and an increased quantity; a subsidy policy continues to subsidize at an excessively high rate relative to cost. For positive shocks, MB* lies between I_P and B. Note that I_P is asymptotic to the line $p+q = 1$, which is parallel to MB*, so the two will never intersect for positive values of θ. In this range a budget policy results in too much abatement and a subsidy policy in too little, but the budget policy is more efficient.

Comparing the budget and quantity instruments, Figure 2 shows a symmetric situation to the subsidy versus budget case. For positive cost shocks, both budget and quantity policies result in too much abatement, but less so in the budget case: MB* is always beneath I_Q and also beneath B. For negative shocks, the asymptotes for I_Q again ensure that I_Q and MB* cannot intersect. Here the quantity policy results in too little abatement, while firms abate too much under the budget policy. The budget policy, however, is more efficient than the quantity policy.

Under uncertainty, the budget policy offers an intermediate outcome between the other policies. It also offers an intermediate expected efficiency outcome:

Proposition 2: Suppose the marginal benefit curve makes a subsidy policy dominate a quantity policy. Then the budget policy also dominates a quantity policy. When a quantity policy dominates a subsidy policy, a budget policy also dominates a subsidy policy.

Either a subsidy or quantity policy can dominate a budget policy. When marginal benefits are perfectly inelastic, the marginal benefit curve will coincide with the optimal quantity constraint (line Q_0) for every marginal cost outcome. In this case, a quantity policy is the first-best outcome for all values of θ: marginal cost and marginal benefits are equated and no efficiency loss occurs. Similarly, when marginal benefits are perfectly elastic, the marginal benefit line coincides with the optimal subsidy constraint, line P_0, and a fixed subsidy rate is efficient. In these extreme cases, budget policy cannot be the best policy. Furthermore, a budget policy continues to be worse when the marginal benefit line is reasonably steep (quantity policy dominates) or reasonably flat (subsidy policy dominates),. But Proposition 2 implies that the budget policy can never

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[18] Unless otherwise stated, we assume that the different policies yield interior solutions for the range of

be the worst policy of the three. The precise conditions are stated in Proposition 3:

Proposition 3: For all marginal benefit curves with slopes $\{m|m \leq -3\}$, a quantity policy dominates a budget policy. For all marginal benefit curves with slopes $\{m| m \geq -1/3\}$, a subsidy policy dominates a budget policy.

Indeed, the *only* marginal benefit line for which budget policy is always superior, irrespective of the distribution of θ, is the case of MB* discussed in Proposition 1. When the marginal benefit curve is close to, but not identical with, MB*, the ordering among the top two policies can, in principle, switch.

Figure 3 illustrates the issue. As before, MB* is tangent to curve B at point E, making the budget policy best. Now consider MB_1, which is slightly steeper than MB*, so that its slope is less than -1 but greater than -3. Budget policy will continue to dominate quantity policy for positive cost shocks, as MB_1 always lies beneath I_Q. But, for sufficiently negative shocks to the marginal cost of abatement, line MB_1 must eventually intersect I_Q., say, for $\theta = \theta_L$. At θ_L, the budget and quantity policies yield identical efficiency losses. For $\theta > \theta_L$, the budget policy dominates; for $\theta < \theta_L$ the quantity policy dominates. In this case, marginal benefits of abatement are so low that the excess abatement under budget policy is inefficient relative to the shortfall in abatement induced by a quantity policy.

Which policy yields a smaller expected efficiency loss depends on the distribution of θ. If uncertainty is sufficiently limited – for example, if θ_L is a lower bound for the feasible values of θ - then the budget policy dominates. However, there exist distributions of θ which, notwithstanding symmetry requirements, give sufficient weight to very low values of θ so that the quantity policy is better.

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feasible marginal costs.

The reverse pattern holds for the relative attractiveness of the subsidy policy and the budget policy, illustrated in Figure 4. Here, problems for budget policy occur when there is a positive shock to cost. The marginal benefit curve MB_2 is somewhat flatter than MB*, and must intersect I_P at a positive value θ^H. Whenever the distribution of θ is bounded above by θ^H, the budget policy dominates the subsidy policy for MB_2. But there exist symmetric unbounded distributions for θ such that a fixed subsidy rate is better. The flatter is the marginal benefit curve, the more restrictive are conditions for f(θ) such that a budget policy dominates a subsidy policy.

These results are summarized in Propositions 4 and 5.

Proposition 4: Let $t \varepsilon [1, 3)$. Then for all marginal benefit curves with slopes $\{m | -t \leq m \leq -1/t\}$ there exist values $\theta^L(t) = -\theta^H(t)$ such that if θ is bounded by $(\theta^L(t), \theta^H(t))$ then a budget policy is the best of the three policies. Furthermore,

(4.1) $\partial\theta^H(t)/\partial t < 0$ and

(4.2) as $t \rightarrow 3$, $\theta^H(t) \rightarrow 0$.

Proposition 5: For each marginal benefit curve with slope $\{m | -1 < m \leq -1/3\}$, there exists a symmetric distribution f(θ) such that E(f(θ)) = 0 and a subsidy policy is better than a budget policy. For each marginal benefit curve with slope $\{m | -3 \leq m < -1\}$, there exist symmetric distributions f(θ) such that E(f(θ)) = 0 and a quantity policy is better than a budget policy.[19]

These results underscore the constraint inherent in the budget policy: the budget represents both a ceiling and a floor. For both very high and very low marginal cost, the budget policy no longer provides useful flexibility for trading off subsidy rates for

[19] These results depend on the existence of positive interior solutions for each regulatory regime for sufficiently large values of θ.

quantities. When marginal cost is low, the marginal cost line intersects the budget curve in the region where the budget curve is asymptotic to the x-axis. Here, the budget policy acts much like the subsidy policy, although the per-unit subsidy is set lower under the budget policy. Because the budget policy requires that the pre-determined budget be spent, firms abate too much.

Alternatively, when the marginal cost is high, the marginal cost line intersects the budget curve in a region where the budget curve is asymptotic to the y-axis. Here, the budget policy resembles the quantity policy: spending the allocated budget leaves no flexibility to reduce the firm's activity to reflect the far lower marginal benefit of abatement. Like the quantity policy, the budget policy results in excessive abatement.

4 Optimal budget policy

The previous sections compared three policies, keeping the expected spending by government constant.[20] However, if we can do better with a budget policy for the same expected cost, we can continue to obtain an advantage from it with a somewhat smaller cost. This section investigates the characteristics of the optimal size of the budget under the budget policy. Our proofs address the special case where the marginal benefit curve has slope -1; the intuition, however, suggests that the results hold more broadly.

As Figure 1 shows, the budget curve lies above the marginal cost curve everywhere that θ is not zero. Thus, given any uncertainty in marginal costs, a budget equivalent to expected costs under the optimal subsidy policy is too large most of the time. Proposition 6 addresses this bias and the existence of an optimal budget.

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[20] The expected cost to the government under an optimal price policy is equivalent to expected spending to industry under optimal quantity regulation, although actual expenditures vary once uncertainty over the marginal cost curve is resolved. See Weitzman (1974).

Proposition 6: Let the marginal benefit curve be defined by $\{(q, p)| q + p = 2\}$; define the marginal cost curves by $\{(q, p)| q - p = \theta\}$. Then a unique budget B* exists that minimizes the expected welfare loss for a budget policy, and where:

$$0 \leq B^* < 1 \text{ whenever } f(\theta) > 0 \text{ for some } \theta \neq 0.$$

Figure 1 also shows that the budget curve and the marginal benefit line diverge as θ moves from zero, suggesting that the optimal budget declines when "uncertainty" – here, the variance of the distribution of marginal cost curves - increases.

Consider the family of probability distributions $g_\lambda(\theta) = (\lambda)f(\lambda\theta)$. Let σ^2 be the variance of f(θ). For each $\lambda > 0$, $g_\lambda(\theta)$ is a symmetric distribution with mean 0 and variance $(1/\lambda)^2\sigma^2$. We equate an increase in uncertainty with a decrease in λ, and consider how the optimal budget, B*(λ), changes with λ.

Proposition 7. Let the marginal benefit curve be $\{(q, p)| q + p = 2\}$. Let $g_\lambda(\theta)$ denote the distribution of marginal cost curves. The welfare-maximizing budget under a budget policy increases with λ; thus the optimal budget B* decreases with the variance of the distribution of marginal costs.

In comparing budget policy with subsidy and quantity policies, we showed that for large positive or negative shocks to the marginal cost of abatement, firms abate too much under a budget policy when the budget is equivalent to expected costs on the other programs. Another interpretation is that the budget is too high, which is established in these propositions for general categories of probability distributions. In brief, for both very low and very high marginal costs, the budget is too high. When these outcomes become more probable – in, as the proposition shows, a mean-preserving spread of the distribution of θ - a lower budget is in order from the outset.

5. Conclusion

For modestly sloped marginal benefit and marginal cost lines, a policy that relies on a pre-set budget may perform better than policies that rest on an *ex ante* establishment of a subsidy or quantity level. But, while uncertainty over costs does not affect the relative efficiency ranking between a subsidy policy and a quantity policy, the extent of uncertainty over marginal costs is critical to comparing the budget policy to other policies. As the marginal benefit line becomes flatter, budget policy will remain preferable to the quantity policy. But only when the plausible range of marginal costs is fairly narrow is a budget policy more attractive than a subsidy policy. As the marginal line becomes steeper, budget policy similarly dominates quantity policy only for increasingly limited distributions over θ. In general, greater uncertainty over costs narrows the set of marginal cost and marginal benefit curves for which a budget policy dominates either or both of the other options.

We examined the optimal size of a budget policy, deriving two results. First, we showed that budget policy can be more efficient that the alternative policies, and that the optimal budget is less than expected government spending under the subsidy policy. Second, we showed that the optimal budget declines when there is greater uncertainty over costs, in the specific sense of a mean-preserving spread in the distribution of marginal cost. The lower expected bill might offset potentially higher costs to the government from the requirement that a budget policy, unlike subsidy and quantity policy, requires *ex post* decisions on either the subsidy rate or the quantity.

The result that greater uncertainty leads to a lower optimal budget is a consequence of our assumption that the bureau spends its entire budget. In fact, the initial reduction in the budget is efficient only because of the bureaucratic imperative to spend all its budget, even when unjustified. Were the bureau willing to spend less than its program budget, or able to do so without penalty – to carry it forward to the next cycle, or reprogram it for a different purpose – then a superior outcome would be reached by allocating the identical budget as expected costs under the subsidy program. A wise bureau could then adjust subsidies or quantities *ex post* to obtain a first-best solution. Indeed, we can think of the expected inefficiency of the policy – the probability weighted difference between the marginal benefit line and the budget constraint – as a measure of bureaucratic inefficiency.

The section 4 implies that the potential attractiveness of budget regulation over the other instruments is broader than we conclude in section 3. By choosing a proper budget – one that takes into account the agency issues involved in bureaucratic control of the program – the budget policy is attractive for a wider range of uncertainty than propositions 4 and 5 conclude. However, for the extreme cases of perfectly inelastic and elastic marginal benefits, and for cases close to these extremes, the alternative instruments remain superior.

Our consideration of a budget constraint can be extended to consider other policies. Some governmental services are provided with a set quantity---think of government building public housing or a new bridge, where cost overruns will be covered. At other times, the government sets a price it will provide for a service, with demand determining the quantity provided---entitlements, as with Medicare, fit this approach, as does a state paying public universities a fixed amount for each student enrolled. And often government sets a budget, with an increase in the cost of providing the service met by providing a smaller quantity---maintenance of highways, parks, and so on meets this approach. We can then ask which approach is more efficient given uncertainty about the costs of providing the service or the benefits of the service. Our analysis suggests how to analyze such issues.

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Appendix

Lemma 1: For a given realization θ of the marginal cost curve, let X denote the outcome for a cap-and-trade system, let Y denote the outcome when government budgets, and let Z denote the outcome when government subsidizes. Then for the marginal benefit curve $MB_Q(\theta)$ which bisects the line segment XY, the efficiency losses under a quantity policy and a subsidy policy are the same.

Proof:

Refer to Figure A1. By construction, EX and YV are parallel and the line segments $XI_Q(\theta)$ and $YI_Q(\theta)$ are equal length. As a result, the EYVX must be a parallelogram and the identified triangles of equal size. Shifting the Marginal Benefit line towards the line $q = Q_0$ yields a welfare advantage to quantity regulation over budget regulation, and vice versa.

Lemma 2: Derivation of I_Q and I_P:

We establish the properties of I_Q; an equivalent construction holds for I_P.

The hyperbola with asymptotes $q + p = 1$ and $q = ½$ which contains the point (1, 1) is given by the formula:

(A1) $I_Q = (q – ½)(q + p - 1) = ½$

By Lemma 1, it is sufficient to show that this curve bisects the line segment of $p – q – \theta = 0$ between the line $q = 1$ and the hyperbola $qp = 1$ for all θ.

Consider a particular value of θ. The q-coordinate for the quantity outcome (X on Figure A1) must be 1, and we set the q-component for the bureaucratic outcome (Y) at $1 + w$ and that for the S, intersection of the marginal cost line and I_Q, at $1 + u$. It is sufficient to show that $w = 2u$.

Let p_Q, p_B , p_S be the p-coordinates for X, Y and S:

$p_Q = 1 + \theta$

$p_B = 1 + w + \theta$

$p_S = 1 + u + \theta$

As Y lies on the hyperbola pq = 1,

(A2) $p_B q_B = 1 = ((1 + w)(1 + w + \theta)$

S lies on the hyperbola $(q - ½)(q + p - 1) = ½$. Substituting:

$$½ = (1 + u - ½)(1 + u + 1 + u + \theta - 1) = (u + ½)(2u + \theta + 1)$$

Thus:

(A3) $1 = (2u + 1)(2u + \theta + 1)$

Comparing (A2) and (A3) establishes that w = 2u.

Proof of Proposition 1

If the price and quantity regulation yield equal expected efficiency, the marginal benefit curve, MB*, equals $\{(q, p)| q + p = 2\}$, and has a slope of -1. As $B = \{(q, p)| qp = 1\}$, this line is also tangent to the curve B at point E.

Consider budget and quantity regulation. We show first that MB* intersects I_Q only at $\theta = 0$. The slope of MB* is –1; the slope of the tangent line to I_Q at point E is -3. As I_Q is a hyperbola, the slopes of the tangents to I_Q increase from $-\infty$ (at the left asymptote) to -1. For negative values of θ, curve I_Q asymptotes to the line q+p=1, which is parallel to, but below, MB*. Thus, MB* and I_Q cannot intersect for non-zero values of θ.

When $\theta = 0$, line MB* intersects I_Q, and so the two regulatory methods have the same efficiency. For all non-zero values of θ, line MB* lies on the same side of I_Q as B (refer to figure 2); therefore, budget regulation is more efficient than quantity regulation.

Similarly, MB* intersects I_P only when $\theta = 0$. For all other values of θ, line MB* lies between on the same side of I_P as B, so that budget policy dominate a price policy.

Proof of Proposition 2

Consider a marginal benefit curve MB^1 that is steeper than MB*, so that its slope is less than -1. In this case, a quantity policy dominates a price policy. But, as discussed in the proof to Proposition 1, the tangencies to I_P always exceed -1, so MB^1 intersects I_P only once (at point E), and otherwise lies between I_P and Q_0. In this case, for all non-zero θ, line MB^1 is steeper than $MB_P(\theta)$, and thus a budget policy dominates a price policy. Similarly, consider a marginal budget curve MB^2 which is flatter than MB*. Then a price policy dominates a quantity policy. But in this case, MB^2 must lie between I_Q and P_0, so that a budget policy dominates a quantity policy.

Proof of Proposition 3

It suffices to prove that quantity regulation dominates budget regulation when the slope of the marginal budget curve is -3, as the result then holds for any steeper marginal benefit curve. By equation A1, MB^Q is tangent to I^Q at (1, 1); hence, budget regulation dominates quantity regulation for $\theta > 0$ and quantity regulation dominates budget regulation for $\theta < 0$.

Let $\theta^H = - \theta^L$.

Define the following lines, shown in Figure A2:

The quantity regulation line: $Q_0 = \{(q, p)| (q - 1) = 0\}$

The tangent to I_Q at (1, 1): $MB^Q = \{(q, p)| (p - 1) + 3(q - 1) = 0\}$

The tangent to B at (1, 1): $MB^* = \{(q, p)| (p - 1) + (q - 1) = 0\}$

The marginal cost line: $MC(\theta^L) = \{(q, p)| (p - 1) - (q - 1) = \theta^L\}$

The marginal cost line: $MC(\theta^H) = \{(q, p)| (p - 1) - (q - 1) = \theta^H\}$

And let:

X = the efficiency loss under quantity regulation for θ^H

Y = the efficiency loss under quantity regulation for θ^L

U = the efficiency loss under budget regulation for θ^H

V = the efficiency loss under budget regulation for θ^L

As $f(\theta)$ is symmetric, it suffices to show that $(V - Y) > (X – U)$

i. The efficiency loss under quantity regulation is identical for θ^H and θ^L. As Figure A3 illustrates, triangles X and Y must be similar.

ii. The outcomes at the intersection of the marginal cost curves and line MB*, shown in Figure A3 as triangle Z for θ^L and triangle W for θ^H, yield identical efficiency losses under quantity regulation; that is, the areas of triangles W, Z, X and Y are congruent. It is tedious but straightforward to show that MB^Q bisects the segment of $MC(\theta^L)$ between Q_0 and MB*, and that MB^Q bisects the segment of $MC(\theta^H)$ between Q_0 and MB*.

iii. As $MC(\theta^L)$ and $MC(\theta^H)$ are parallel to, and equidistant from MC(0), the major axis of hyperbola B, the distances from MB* to B on lines $MC(\theta^L)$ and $MC(\theta^H)$ (labeled d and f on Figure A2) are identical. The efficiency loss under budget regulation for $MC(\theta^L)$ exceeds triangle Z by the additional area G, while the efficiency loss under budget regulation for $MC(\theta^H)$ is reduced from triangle W by area H. But as shown in Figure A2, H must be smaller than G. Thus:

$$(V - Y) = Z + G – Y > X – (W – H) = (X – U)$$

Proofs of Proposition 4 and 5

Propositions 4 and 5 follow directly from figures 3 and 4, and the construction in Proposition 3. Consider the budget-quantity comparison. MB^Q is tangent to curve I_Q at point E. Any marginal benefit curve with slope between (-1) and MB^Q must intersect I_Q

for some negative value of θ. Quantity regulation then will dominate for low costs. As long as costs are constrained to be higher than this intersection (i.e., θ is bounded), budget regulation dominates. But consider a distribution for θ that places virtually all its weight on the extreme values of θ. In this case, Proposition 3 holds, as the advantage of quantity regulation over budget regulation at low marginal costs of abatement outweighs the advantage of budget regulation over quantity regulation at high costs.

Proof of Proposition 6

We first change variables to rotate the marginal benefit curve and marginal cost curves making them parallel the s and t axis. The same change in variables reorients the hyperbola perpendicular to the s-axis, so that for each θ, the distance d between the intersection of the marginal cost curve and hyperbola B (i.e., the outcome of budget regulation) and the marginal benefit curve can be easily calculated from the equation of the hyperbola, and the welfare loss associated with θ is ½ d^2. Let:

$$s = q - p; \quad t = q + p$$

Note that s = θ; we use this notation in the remainder of the proofs for this section.

Then:

$$q = (s + t)/2 \text{ and } p = (t - s)/2$$

$$B = \{(s, t) | ((s+t)/2)((t - s)/2) = B\} \Rightarrow B = (t^2 - s^2)/4$$

The equation for the budget constraint is given by:

$$t(s; B) = (4B + s^2)^{1/2}$$

Define:

$$d(s; B) = t(s; B) - t(0; 1) = (4B + s^2)^{1/2} - 2$$

The welfare loss associated with s and B is:

$$WL(s; B) = ½\, d^2 = ½\, [(4B+s^2)^{1/2} - 2]^2 = ½[4B + s^2 + 4 - 4(4B + s^2)^{1/2}]$$

$$WL(B; \lambda) = \int WL(s; B)g_\lambda(s)ds = 2B + \lambda^2\sigma^2 + 2 - 2\int[4B + s^2]^{1/2}g_\lambda(s)ds$$

$$\partial WL/\partial B = 2 - 4\int[4B + s^2]^{-1/2}\, g_\lambda(s)ds$$

$$\partial^2 WL/\partial B^2 = 8\int[4B + s^2]^{-1/2}\, g_\lambda(s)ds > 0$$

As $s^2 \geq 0$, at $B = 1$,

$$\partial WL/\partial B = 2 - 4\int[4 + s^2]^{-1/2}\, g_\lambda(s)ds \geq 2 - 4\int[4]^{-1/2}\, g_\lambda(s)ds = 0$$

where the inequality is strict whenever $g_\lambda(s)$ is positive for some non-zero s.

At $B = 0$, $\partial WL/\partial B = 2 - 4\int[s^2]^{-1/2}\, g_\lambda(s)ds = 2 - 4\int|s|g_\lambda(s)ds$. An interior solution is guaranteed as long as this expression is negative. We consider only interior solutions in the subsequent analysis; however, note that for sufficiently small realizations of λ, the optimal budget may be at B == 0.

Let $2 - 4\int|s|g_\lambda(s)ds < 0$. Then we can define an optimal budget:

(A3) $B^*(\lambda) = \{B|\int[4B + s^2]^{-1/2}\, g_\lambda(s)ds = ½\}$

Proof of Proposition 7.

Differentiating (*) and rearranging:

(A4) $dB^*/d\lambda\, [2\int[4B + s^2]^{-3/2}\, g_\lambda(s)ds] = \int[4B + s^2]^{-1/2}\, [d(g_\lambda(s))/d\lambda]ds$

In (A4),$dB^*/d\lambda$ is multiplied by a positive factor, hence it is sufficient to show that the right-hand side of (A4) is positive.

(A5) $g_\lambda(s) = \lambda f(\lambda s) \rightarrow d(g_\lambda(s))/d\lambda = f(\lambda s) + \lambda s f'(\lambda s)$

Substituting from (A5) and (A3):

$\int[4B + s^2]^{-1/2}\, [d(g_\lambda(s))/d\lambda]ds = \int[4B + s^2]^{-1/2}\, f(\lambda s)ds + \int[4B + s^2]^{-1/2}\, \lambda s f'(\lambda s)$

$= (1/\lambda)\int[4B + s^2]^{-1/2}\, \lambda f(\lambda s)ds + \int[4B + s^2]^{-1/2}\, \lambda f'(\lambda s) = (1/2\lambda) + \int[4B + s^2]^{-1/2}\, \lambda s f'(\lambda s)$

We integrate by parts the final term, letting

$$u = (4B + s^2)s;\quad du = (-1/2)\, s(4B+s^2)^{-3/2}2sds + (4B+s^2)^{-1/2}ds$$

$$v = \lambda\, f'(\lambda s)ds;\quad dv = f(\lambda s)$$

With appropriate substitutions:

$dB^*/d\lambda\ [2 \int[4B + s^2]^{-3/2}\, g_\lambda(s)ds = 1/(2\lambda) - \int[4B + s^2]^{-3/2}\, 4B\, g_\lambda(s)(1/\lambda)ds$

$= 1/(2\lambda) - \int[4B + s^2]^{-1}[4B + s^2]^{-1/2}\, 4B\, g_\lambda(s)(1/\lambda)ds$

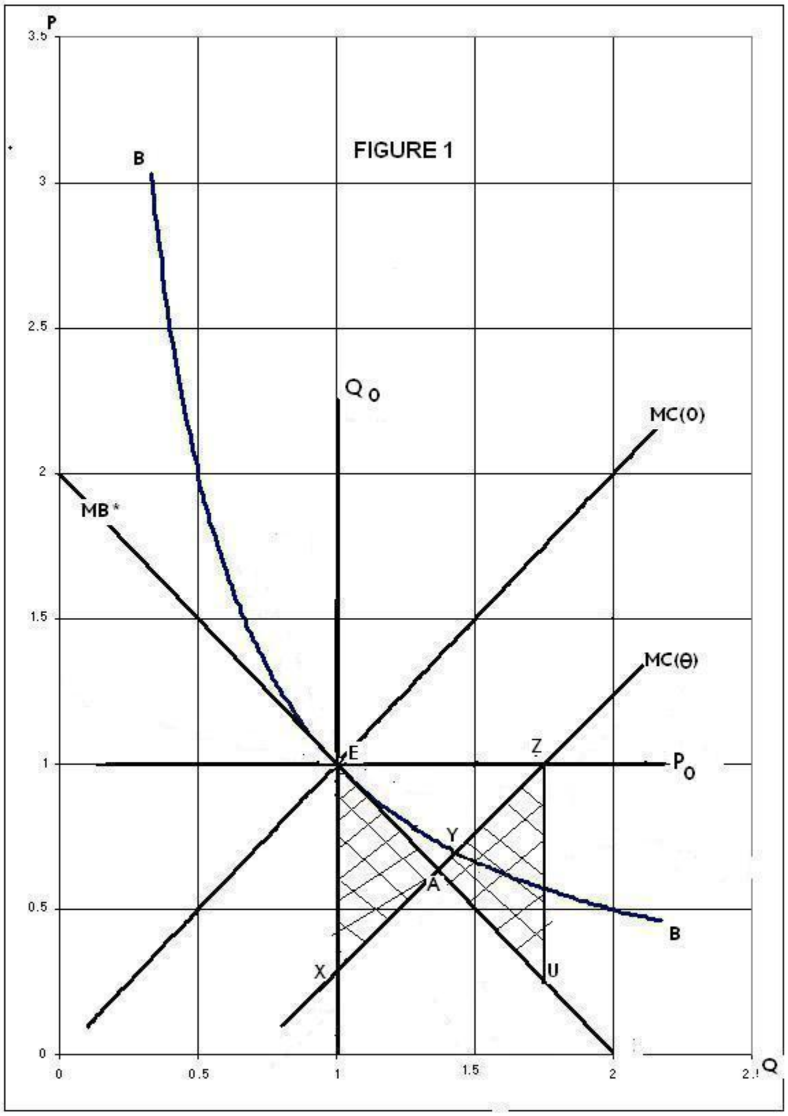
As $s^2 > 0$ for all $s \neq 0$,

$1/(2\lambda) - \int[4B + s^2]^{-1}[4B + s^2]^{-1/2}\, 4B\, g_\lambda(s)(1/\lambda)ds >$

$1/(2\lambda) - (1/\lambda) \int[4B]^{-1}[4B + s^2]^{-1/2}\, 4B\, g_\lambda(s)ds = 1/(2\lambda) - (1/\lambda) \int[4B + s^2]^{-1/2} g_\lambda(s)ds$

By (A3) the final term in the preceding equation equals (1/2), so the expression is positive and

$dB^*/d\lambda > 0.$


P
FIGURE 1
B
MB*
Q0
MC(0)
MC(θ)
E
Z
P0
Y
A
X
U
B
Q
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5

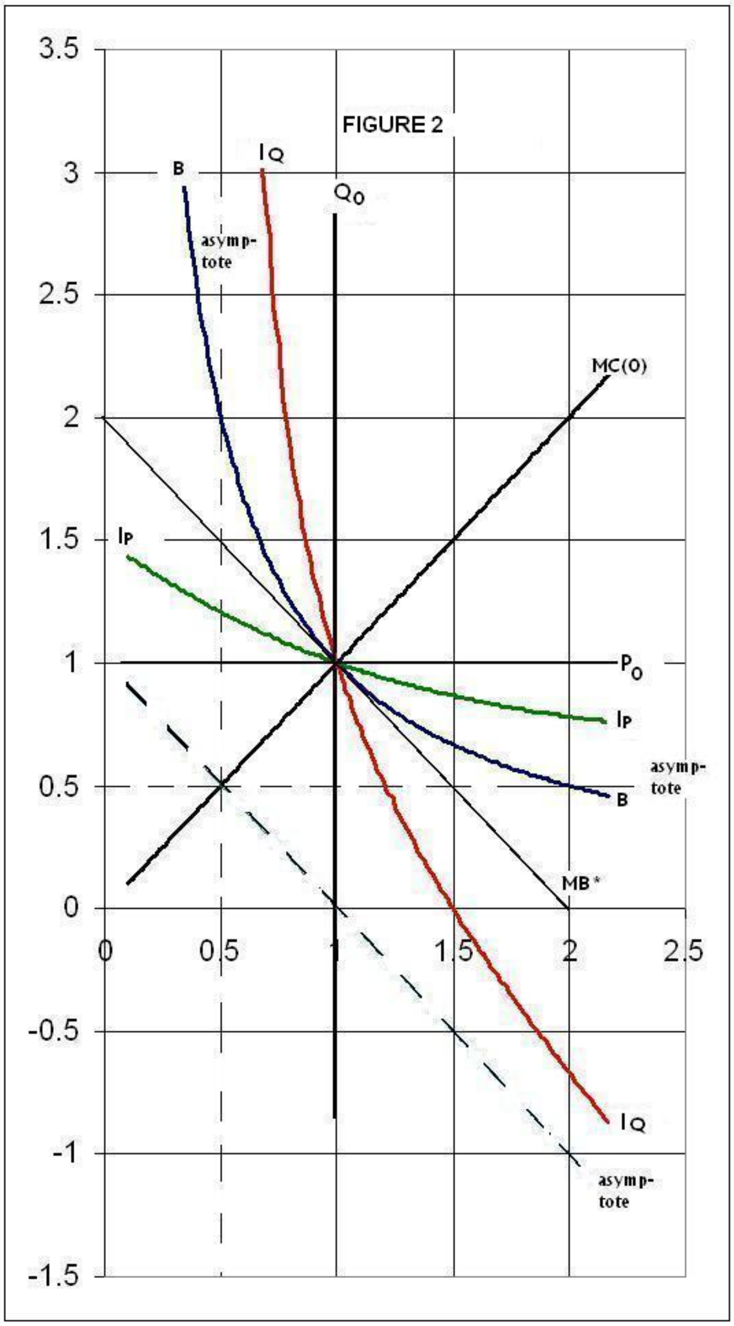


FIGURE 2
B
IQ
QO
asymp-
tote
MC(0)
IP
PO
IP
asymp-
tote
B
MB*
IQ
asymp-
tote
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5

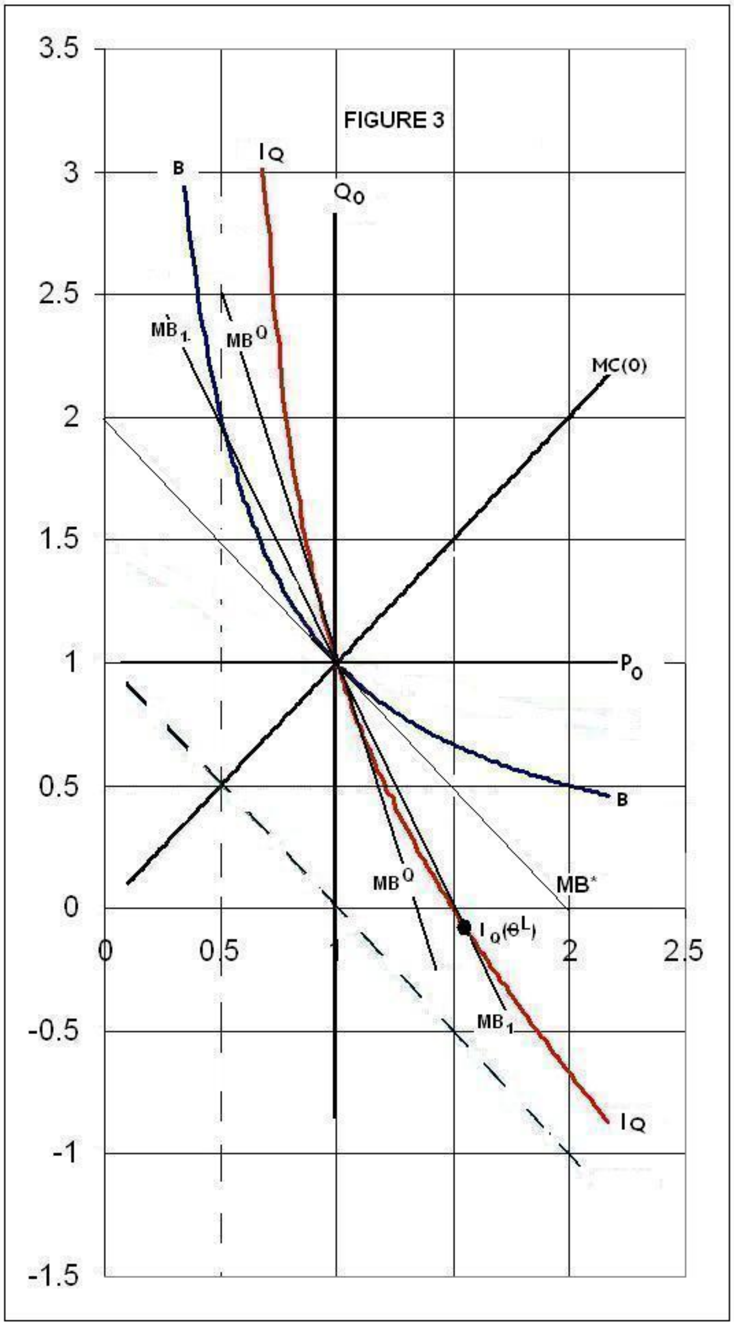


FIGURE 3
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.5
1
2
2.5
B
IQ
Q0
MB1
MB0
MC(0)
P0
B
MB0
MB*
IQ(θL)
MB1
IQ

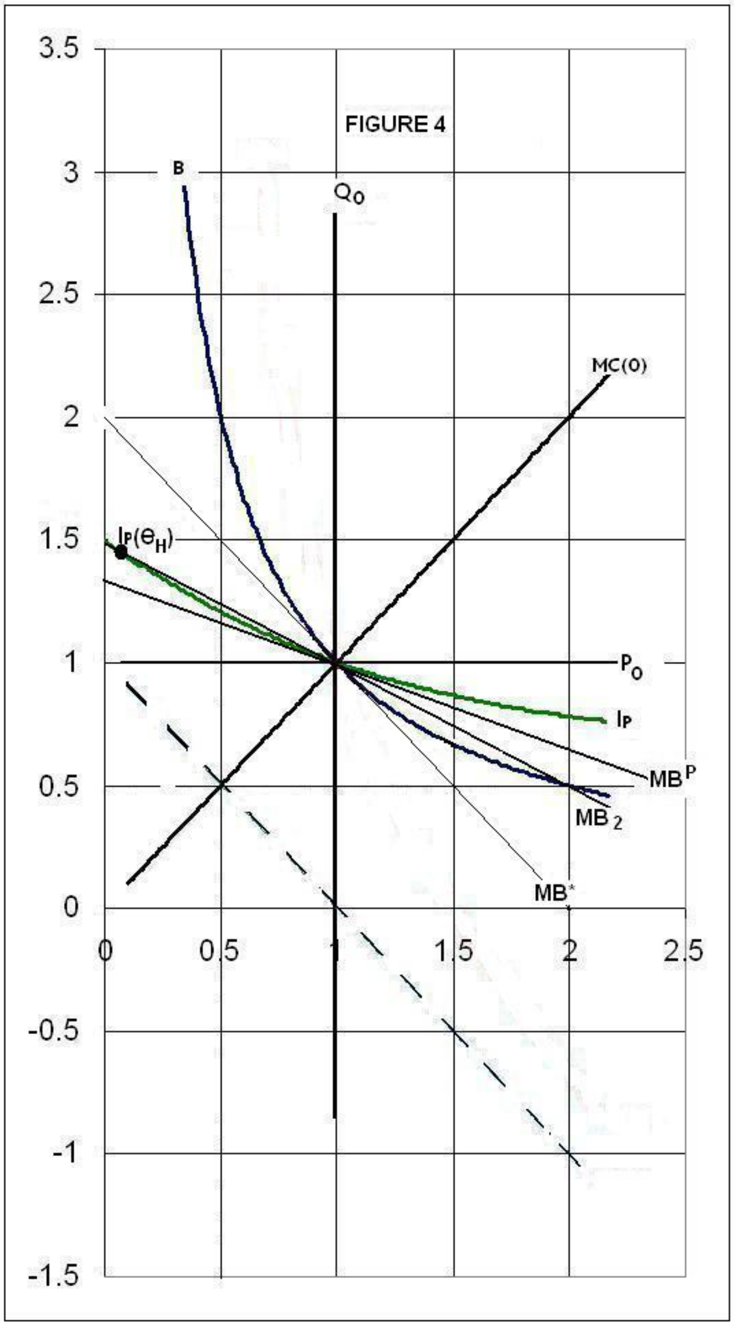


FIGURE 4
B
Q0
MC(0)
IP(eH)
P0
IP
MBP
MB2
MB*
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5

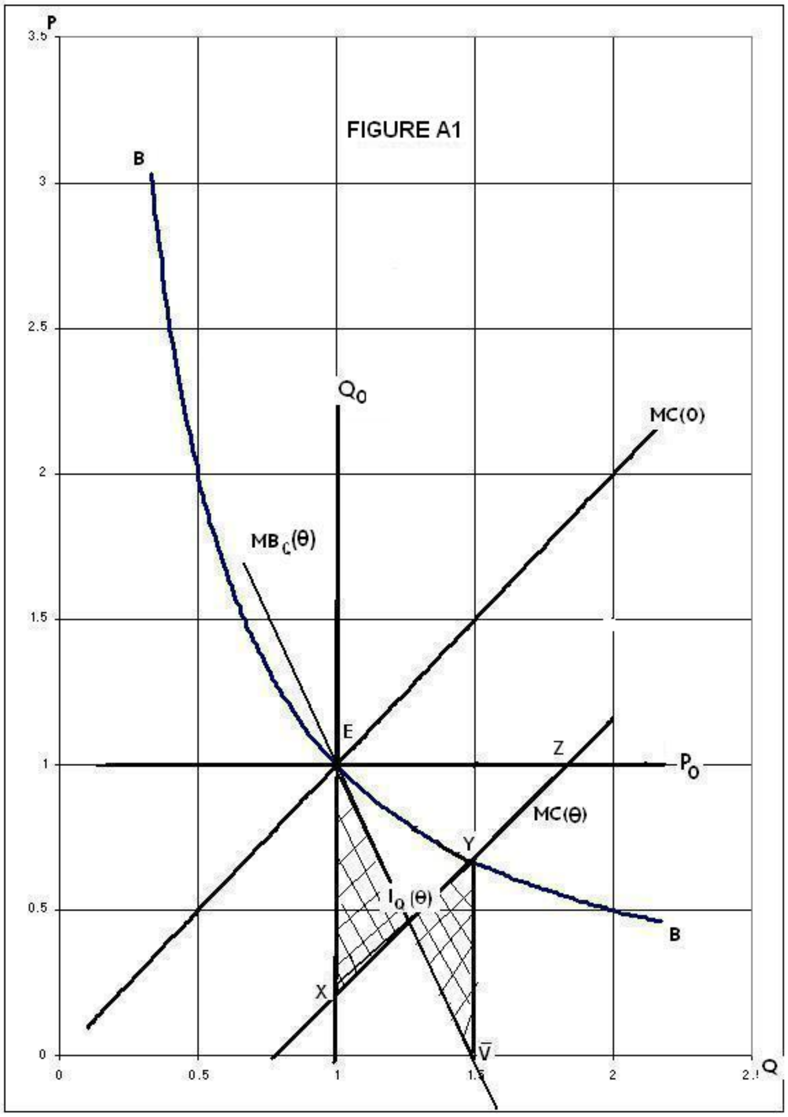


FIGURE A1
P
Q
B
B
Q_0
MC(0)
$MB_C(\theta)$
E
Z
P_0
MC(θ)
Y
$I_0(\theta)$
X
$\bar{V}$
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5

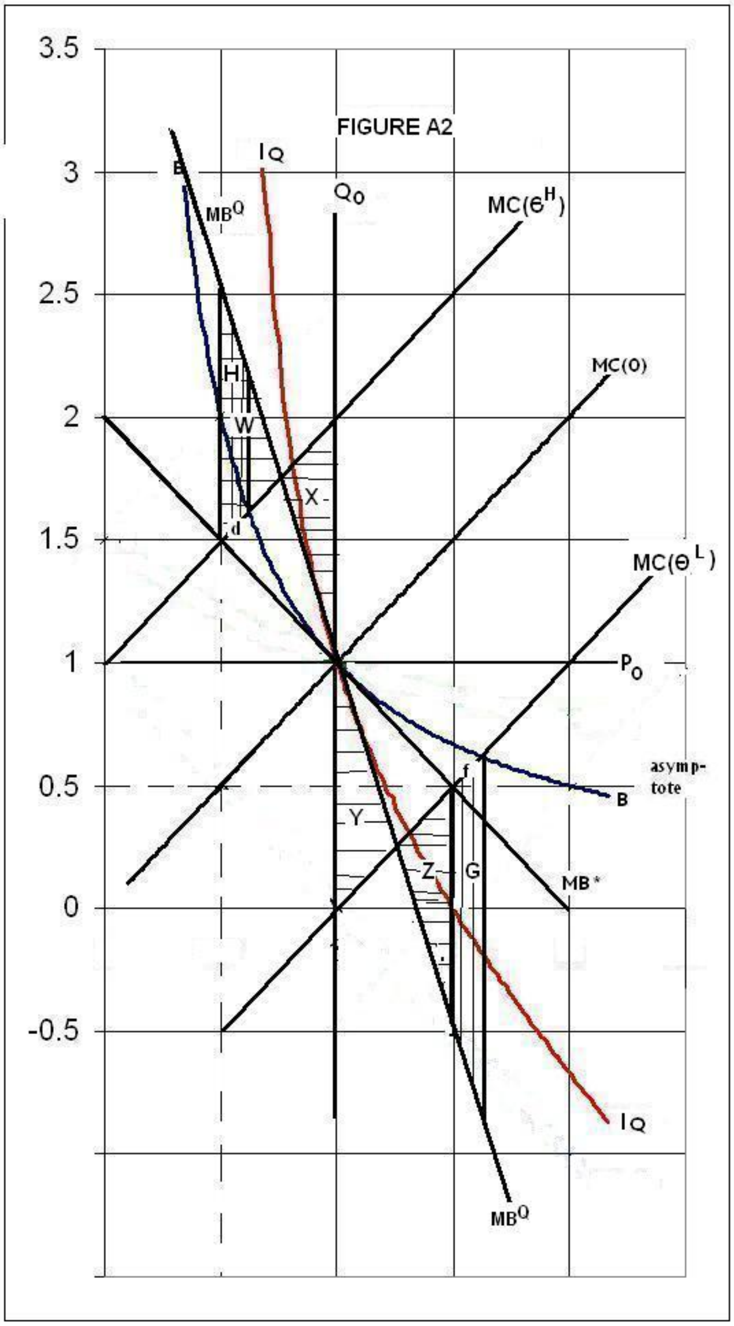


FIGURE A2
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
IQ
Q0
B
MBQ
MC(θH)
MC(0)
MC(θL)
H
W
X
d
P0
asymp-
tote
B
f
Y
Z
G
MB*
IQ
MBQ