Competition and Offshoring

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Abstract

I present a model of offshoring decisions with heterogeneous firms, random adjustment costs, and endogenous markups. The model departs from the conventional view of self-selection of more productive firms into offshoring. Instead, by characterizing the offshoring decision as a lumpy investment decision subject to heterogeneous adjustment costs, the model obtains an inverted-U relationship between firm-level productivity and the probability of offshoring; hence, the most productive firms are less likely to offshore than some lower-productivity firms. In this setting, a tougher competitive environment has two opposing effects on firm-level offshoring likelihood: a Schumpeterian effect—accounting for the negative effect of competition on offshoring profits—and an escape-competition effect—accounting for the effect of competition on the opportunity cost of offshoring.

Keywords: competition, offshoring, heterogeneous firms, endogenous markups, adjustment costs.
JEL codes: F12, F23.

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1 Introduction

Changes in the competitive environment affect firms' pricing, production, and innovation decisions. A tougher competitive environment, for example, reduces firms' markups, causes some firms to exit, and drives some other firms to take bold actions to improve their production processes to remain competitive. Offshoring—the international fragmentation of production—is a particular type of process innovation that has gained considerable importance in recent years.[1] The objective of offshoring is simple: to reduce a firm's marginal cost by moving a part of its production process to another country with lower factor prices. But if offshoring implies lower marginal costs, who has more incentives to offshore? A high-productivity firm or a low-productivity firm struggling for survival? The offshoring decision implies a production-process adjustment that resembles an investment decision. As such, it is subject to disruption costs that are likely to be larger for high-productivity (and high-profit) firms; hence, the offshoring incentives for a high-productivity firm may be less than the incentives for a lower-productivity firm.

This paper provides a new framework to study the effects of competition on firms' offshoring decisions in a setting in which low-productivity offshoring firms coexist with high-productivity non-offshoring firms. At its core, my model characterizes the offshoring decision as a lumpy investment decision subject to heterogeneous—across firms and over time—adjustment costs. The model shows an inverted-U relationship between firm-level productivity and offshoring probability: a more productive firm is not necessarily more likely to offshore than a less productive firm. In this context, an increase in the level of competition has two opposing effects in the offshoring likelihood of firms. On the one hand, more competition decreases profits of offshoring firms, giving non-offshoring firms less incentives to alter their production processes. On the other hand, although profits of both offshoring and non-offshoring firms might decline, their difference—the incremental profits from offshoring—may increase, making offshoring more attractive relative to non-offshoring. Following the terminology of Aghion *et al.* (2005) in their competition and innovation analysis, I refer to the negative force as the *Schumpeterian effect*, and to the positive force as the *escape-competition effect*. The escape-competition effect is well-known in the economics of innovation literature, but has not been considered in offshoring models or in settings with heterogeneous firms.

This paper departs from the conventional approach in heterogeneous-firm offshoring models of basing offshoring decisions on homogeneous (independent of productivity) fixed costs. In those models, as with the exporting decision in the model of Melitz (2003), only the most productive firms offshore—there exists a cutoff productivity level that separates offshoring from

[1] Offshoring is revolutionizing international trade. For goods, it is now common to find final manufactured goods whose production is fragmented in two or more countries (in terms of value of trade, cars and computers are the leading examples); for services, it is now common to receive customer support from call centers located in other countries (from airlines lost-baggage tracking to financial services). In general, offshoring involves an advanced source country and a developing destination country, with the difference in labor prices as the main force driving offshoring decisions. The extent of offshoring in total trade value is significant. Consider, for example, the cases of China and Mexico—the first and third top exporters to the U.S., respectively. For these countries, offshoring activities (*e.g.* assembly) account for about 50 percent of their total exports (see Yu, 2011 for China, and Bergin, Feenstra and Hanson, 2009 for Mexico).

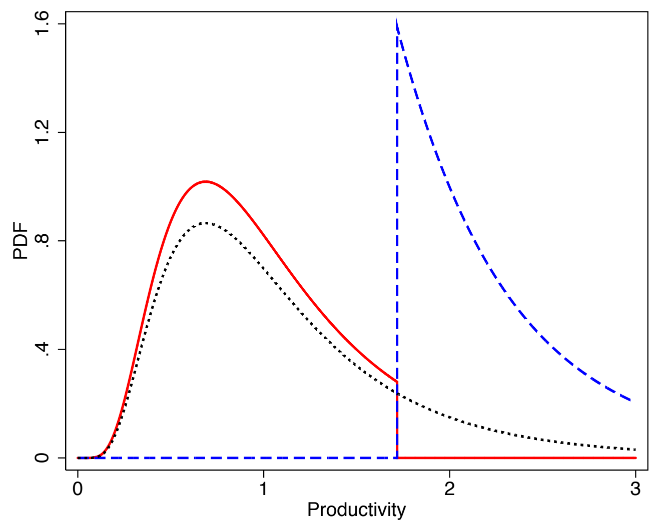


Figure 1: Productivity distributions in a model with homogeneous fixed costs of offshoring: all firms (*dotted*), purely-domestic firms (*solid*), and offshoring firms (*dashed*)

non-offshoring firms.[2] Thus, as shown in Figure 1, homogeneous-fixed-cost models predict a right truncation in the productivity distribution of purely-domestic firms, and a left truncation in the productivity distribution of offshoring firms.[3] As with exporting, very few firms offshore and therefore, the truncation of the distributions should occur at a high productivity level. In the U.S., for example, Bernard *et al.* (2007) report that only 14% of manufacturing firms were involved in importing activities in 1997, which then implies that—if the data satisfies the homogeneous-fixed-cost assumption—evidence of truncation for U.S. manufacturing firms should appear in the last quintile of the productivity range.

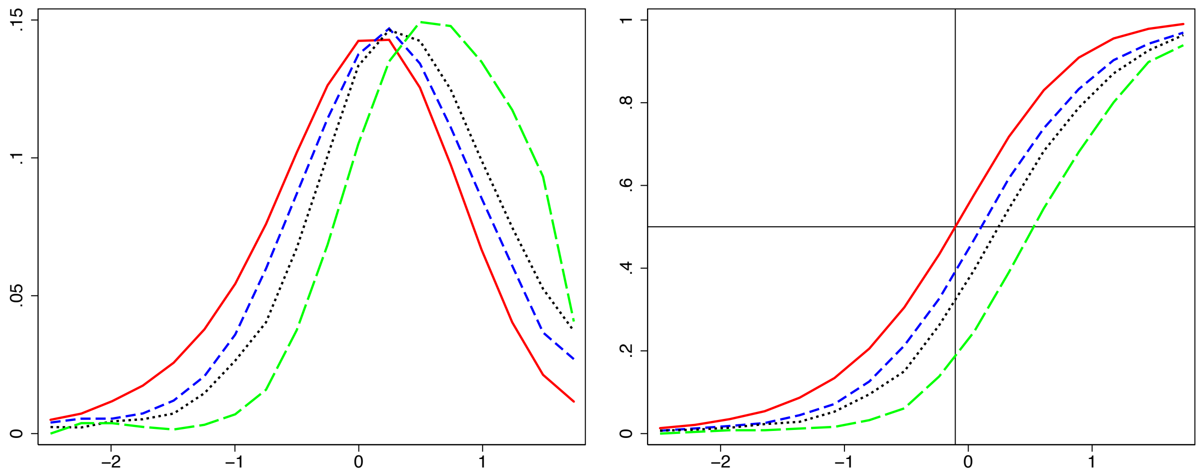
Evidence from Japan, however, suggests a different story for the productivity distributions of non-offshoring and offshoring firms. From Tomiura (2007), Figure 2 shows the empirical log productivity distributions (compared against each firm's industry mean) of purely-domestic and offshoring Japanese manufacturing firms in 1998.[4] There are two organizational types of offshoring: foreign outsourcing (arm's-length trade) and vertical foreign direct investment (related-party or intra-firm trade).[5] Figure 2 shows productivity distributions for offshoring firms that *(i)* do outsourcing, but not FDI or exporting, *(ii)* do FDI, but not outsourcing or exporting, and *(iii)* do outsourcing, FDI, and also export (these are the *most globalized* firms). The empirical probability density functions show no evident signs of the truncation implied by homogeneous-fixed-cost models—with only 5% of Japanese manufacturing firms involved in

[2]For example, the model of Antràs and Helpman (2004) sorts firms into different offshoring organizational structures based on a homogeneous fixed cost for each structure. In that model, the least productive firm keeps all its production activities inside the firm in a single location (the firm is not able to cover the fixed cost for any of the other production structures), while the most productive firm vertically integrates its production process across international borders.

[3]Figure 1 assumes a log normal distribution of productivity for all firms. Although the Pareto distribution is frequently used in heterogeneous-firm models, Combes *et al.* (2012) find that the productivity distribution of French firms is best approximated by a log normal distribution.

[4]I thank Eiichi Tomiura for providing me with the data from his Figures 1, 2, and 3.

[5]In the first type, the offshoring firm subcontracts a part of its production process with an independent foreign firm; in the second type, the offshoring firm owns a subsidiary in a foreign country (see Antràs and Rossi-Hansberg, 2009 for a review of the literature on production organization and trade).



(a) Probability density functions (b) Cumulative distribution functions

Figure 2: Log productivity distributions for Japan's manufacturing firms: purely-domestic (*solid*), outsourcing (*dashed*), FDI-active (*dotted*), and most-globalized (*long-dashed*) — the industry mean is subtracted from each firm's log productivity (Source: Tomiura, 2007)

offshoring activities, the implied truncation should occur in the last decile of the productivity range. To the contrary, there is a high degree of coexistence between high-productivity purely-domestic firms and low-productivity offshoring firms. Moreover, the density functions for the different types of offshoring firms look like right-shifted versions of the density function for purely-domestic firms. This fact is important given the existence of an *offshoring productivity effect*—the decline in marginal cost due to offshoring—for which there is strong theoretical and empirical support.[6] Hence, the ex-ante productivity distributions of offshoring and non-offshoring firms may be very similar, but ex-post, the offshoring productivity effect shifts to the right the distribution of offshoring firms. In that case, the average productivity of offshoring firms is higher precisely because they offshore, not the other way around. As I show below, the model in this paper can generate this result.[7]

The model's main ingredients are firm heterogeneity in productivity, non-convex adjustment costs of offshoring, and endogenous markups. In this framework, more productive firms (with lower marginal costs) charge lower prices, haver larger market shares, and have higher markups.

[6] Grossman and Rossi-Hansberg (2008) present a theoretical discussion of the offshoring productivity effect. Empirically, Amiti and Konings (2007) (for Indonesia) and Goldberg *et al.* (2010) (for India) show evidence of increases in plan-level productivity after reductions in input trade costs.

[7] Although there is strong evidence that firms that participate in international markets—as exporters or importers—are *on average* larger and more productive than firms that only operate domestically (see Bernard, Jensen and Schott, 2009), a truncation of the type suggested in Figure 1 is also absent in the evidence for exporting and non-exporting firms. For example, for U.S. exporters and non-exporters, Bernard *et al.* (2003) show bell-shaped empirical productivity distributions with a substantial degree of overlap: though the exporters' distribution is to the right of the non-exporters distribution, there is a well-established coexistence between low-productivity exporters and high-productivity non-exporters (see their Figure 2B). Similarly, Hallak and Sivadasan (2011) present evidence of the coexistence of large non-exporting firms and small exporting firms in Chile, Colombia, India, and the United States. To explain this fact, they propose a second dimension of heterogeneity: besides productivity, they assume that firms are heterogeneous in "caliber" or quality-production ability. For China, Lu (2010) even finds that the average productivity of exporting firms is lower than the average productivity of non-exporting firms.

Given the model's Melitz-type structure, a cutoff productivity level determines the tradability of differentiated goods; firms with productivity levels below the threshold do not produce. The price set by a firm with a productivity identical to the cutoff level is equal to the marginal cost—its markup is zero. A differentiated-good firm, however, can move a part of its production process to another country to take advantage of lower wages. A firm which offshores has a lower marginal cost, a higher markup, and larger profits.

But if offshoring implies lower marginal costs and larger profits, why don't all firms offshore? Because the offshoring decision is costly. It involves non-negligible relocation and reorganization costs.[8] Therefore, as with an investment decision, the offshoring decision is lumpy. Inspired by the model of Caballero and Engel (1999) on lumpy investment decisions in a generalized (S, s) setting, I introduce random adjustment costs of offshoring. Every period, each non-offshoring firm draws an offshoring adjustment cost from a probability distribution—adjustment costs vary through time and are not necessarily the same for two firms with identical productivity. If the adjustment cost draw is below an endogenously determined threshold, the firm adjusts its production process and begins offshoring. The adjustment cost has two components: one related to the firm's output, and one independent of it. The first component is proportional to the firm's profits and hence, it is more relevant for more productive (and larger) firms—larger firms are likely to face higher reorganization costs. The second component is relatively more important for low-productivity firms, as they have low profits and hence, have lower opportunity costs of reorganization. Under this framework, the relationship between productivity and offshoring probability for non-offshoring firms is non-monotonic.

The model solves for two cutoff productivity levels: one for non-offshoring firms and one for offshoring firms. The latter is smaller than the former, which implies the existence of a range of firms that would not produce if they had not offshored. A tougher competitive environment—driven, for example, by an increase in the substitutability between varieties—increases the cutoff productivity levels for both non-offshoring and offshoring firms, causing lower markups and the exit of the least productive firms. For non-offshoring firms, the maximum adjustment cost they are willing to incur to become offshoring firms also changes, affecting their offshoring probabilities. As a result of the opposite forces of the Schumpeterian and escape-competition effects, the offshoring probability declines for some firms but increases for others. I demonstrate the existence of a level of productivity that separates out non-offshoring firms according to the dominating effect, with the Schumpeterian effect—the cleansing effect of competition—dominating for the least productive firms.

The endogenous-markups structure in this paper helps us to define in a natural and precise way what we mean by a "tougher competitive environment." Given the intimate link between firms' markups and the level of competition in a market, we say that the competitive environment is tougher if every active firm that keeps the same production process is forced to reduce

[8] These costs also include implicit components. Consider, as example, a chip producer in the U.S. semiconductors industry. By producing its chips in the U.S., the firms knows that the intellectual property (IP) rights governing its production process are protected. Although the firm knows that producing its chips in an emerging country would be cheaper, the firm will be reluctant to transfer its production process abroad because of the high risk of IP rights violations.

its markup. Based on this definition, we identify three drivers of a tougher environment: more substitutability between varieties, larger market size, and lower adjustment costs of offshoring. Following Bergin and Feenstra (2000), the model assumes translog preferences to generate endogenous markups. Importantly, the advantages of this variable markups' approach come at no cost in tractability—when compared to a standard constant elasticity of substitution (CES) model.

Though the model's motivation is based on offshoring decisions, this framework can be applied to any type of firm-level decision involving a production-process innovation. In particular, the model adapts naturally to an application on technology upgrading and international trade. In a recent study, Bustos (2011) finds that for Argentinian firms facing tariff reductions from Brazil, most technology-upgrading changes happen in the third quartile of the distribution of firm size. If firm size is positively related to productivity, Bustos's finding implies an inverted-U relationship between firm-level productivity and technology-upgrading likelihood. By modeling the technology-upgrading decision as a lumpy investment decision subject to heterogeneous adjustment costs, this paper provides an appealing explanation for the observed inverted-U relationship.

The paper is organized as follows. Section 1.1 reviews related literature. Section 2 presents the model, with special emphasis in section 2.2, which describes the offshoring decision problem. In Section 3, I present the model's implications for changes in the competitive environment, including a description of the Schumpeterian and escape-competition effects, and a numerical simulation. Section 4 presents a brief technology-upgrading application. Finally, section 5 concludes.

1.1 Related Literature

As background for the model, this section discusses previous literature along two lines. First, after pointing out the similarities between offshoring and investment decisions, we overview the literature on the importance of the type of adjustment costs assumed in my model. And second, we discuss theory and evidence on the Schumpeterian and escape-competition effects in the context of the competition and innovation literature.

Even if not restricted to high-productivity firms, offshoring is a rare activity. As discussed above, this paper accounts for this fact by assuming heterogenous adjustment costs of offshoring. To model these costs, I start from the observation that the offshoring decision is no different to an investment decision: it is discrete, may involve large capital adjustments, and creates disruptions as the firm reorganizes. Therefore, I follow the literature on lumpy investment and rely on non-convex adjustment costs. Empirically, these costs have been shown to be important. In particular, studies using U.S. plant- and firm-level data show that non-convex adjustment costs are necessary to match the dynamics of plant-level investment (Cooper and Haltiwanger, 2006), and output fluctuations after uncertainty shocks (Bloom, 2009). As well, I follow Caballero and Engel (1999)—who also obtain large estimates for non-convex adjustment costs using U.S. data—and add a stochastic element to the offshoring adjustment cost. The stochastic approach not only makes the model tractable, but also adds a new dimension of reality, recognizing that offshoring opportunities present themselves at random, with offshoring adjustment costs differing across firms (even if they are equally productive) and varying over time.

As the opportunity cost of disruptions is likely to be higher for more profitable firms, non-convex adjustment costs in the lumpy-investment literature include a component that increases with a firm's profits. Along the same lines, Holmes, Levine and Schmitz (2012) present a model about the effects of increased competition on technology adoption when costs from switchover disruptions matter. In their model, a monopolist is unwilling to adopt a new technology because of the high opportunity cost of switchover disruptions. The price that the monopolist can charge is, however, limited by the marginal cost of potential rivals. Hence, if that marginal cost falls—in a shock interpreted as an increase in competition—the monopolist's opportunity cost of switchover disruptions also falls, which then may drive the firm to adopt the new technology. Bloom, Romer and Van Reenen (2010) obtain a similar result in their trapped-factors model of innovation.[9,10] There is a close relationship between these competition and innovation models, and my approach to study the effects of competition on offshoring decisions. For example, the inverse relationship between competition and the opportunity cost of innovation obtained in the previous models, is just another version of the escape-competition effect that appears in my model.

The terminology I use to refer to the two effects of competition on offshoring decisions is borrowed from Aghion *et al.* (2005), who find an inverted-U relationship between competition and innovation at the industry level. In their model, the effects of competition on pre-innovation and post-innovation profits depend on whether an industry is leveled (composed of neck-and-neck firms) or unleveled (composed of leaders and followers). In neck-and-neck sectors the difference between pre- and post-innovation profits increases with competition, and hence firms innovate to "escape competition." The opposite happens for laggard firms in unleveled sectors, and hence innovation declines—the Schumpeterian effect of competition. In the end, the industry-level inverted-U shape is generated by changes in the composition of leveled and unleveled sectors in the economy.[11] Though describing similar competition effects, the objective of the model of Aghion *et al.* is to explain the competition-innovation relationship at the industry level, while my model's analysis is intra-industry. More generally, competition and innovation models abstract from firm heterogeneity and endogenous markups. Therefore, in contrast to the model below, they have limited predictions on the relationship between firm-level productivity and innovation

[9]In the model of Bloom, Romer and Van Reenen (2010), production factors are partially trapped in producing old goods because of good-specific sunk investments (*e.g.* learning by doing): if the firm redeploys workers to innovation activities, it loses the human capital related to the production of the old good—a switchover cost. An increase in competition from a low-wage country drives down the firm's profit from the old good, causing a decline in the opportunity cost of innovation, and hence allowing the reallocation of the trapped factors to innovation activities.

[10]The idea that firms' production-process enhancements occur after declines in opportunity costs due to lower profitability has also been used in the context of firms' restructuring decisions during recessions (see, for example, Aghion and Saint-Paul, 1998 and Berger, 2012).

[11]If the initial level of competition is low, the fraction of neck-and-neck sectors is higher, and therefore, the escape-competition effect dominates after an increase in competition, and the innovation rate increases. The opposite happens if the initial level of competition is high: the fraction of unleveled sectors is higher, the Schumpeterian effect dominates after an increase in competition, and the innovation rate declines.

likelihood, the distributions of innovating and non-innovating firms, and the distribution of markups. Moreover, the Melitz-type monopolistic competition framework of my model makes it amenable to extensions and applications in different settings.

Besides presenting the model discussed above and showing substantial evidence on disruption costs, Holmes, Levine and Schmitz (2012) survey the empirical literature on the effects of competition on productivity-enhancing innovations. Based on industry-specific and trade liberalization studies, they observe two general facts: *(i)* competition reduces establishment and industry sizes, and *(ii)* competition spurs establishment-level productivity. These facts are consistent with the interaction of the Schumpeterian and escape-competition effects described in my model. In a related survey, Syverson (2011) mentions similar evidence and highlights the *selection* and *within* effects of competition on aggregate productivity. For an increase in competition, the selection effect refers to the increase in aggregate productivity driven by the reallocation of activity towards more productive firms, while the within effect refers to the increase in aggregate productivity driven by firms making productivity-enhancing decisions. In my model, the dominance of the Schumpeterian effect in low-productivity firms gives rise to selection effects on aggregate productivity, while the dominance of the escape-competition effect in the other firms generates within effects.

2 The Model

This section presents a heterogeneous-firm model of offshoring decisions with endogenous markups and random adjustment costs of offshoring.

I assume a country inhabited by a continuum of households in the unit interval and with two production sectors: a homogeneous-good sector and a differentiated-good sector. Firms in the differentiated-good sector are heterogeneous in productivity. Each household provides a unit of labor at a fixed wage level to any of the sectors in the economy. However, wages differ between this country and the rest of the world. In particular, the wage abroad is below the domestic wage. This fundamental difference gives firms in the differentiated-good sector an incentive to split the production process between the domestic country and the rest of the world. Nevertheless, to begin offshoring, a firm must incur adjustment—or disruption—costs.

First, I specify preferences, obtain the demand, and discuss pricing and production decisions in the differentiated-good sector. Second, I describe the offshoring decision and obtain the key relationship between productivity and offshoring probability. Third, I show the distributions of offshoring and non-offshoring firms and present several results on average prices, market shares, and the composition of sellers. The section concludes with the specification of the free-entry condition that closes the model.

2.1 Model Setup

2.1.1 Preferences and Demand

Households define their preferences over a continuum of differentiated goods and a homogeneous good. In particular, the utility function of the representative household is given by

$$U = q_h + \eta \ln Q, \tag{1}$$

where q_h denotes consumption of the homogeneous good, Q is a consumption index of differentiated goods, and η is a parameter that indicates the degree of preference for differentiated goods. Following Feenstra (2003) and Rodríguez-López (2011), I assume that Q satisfies the symmetric translog expenditure function

$$\ln E = \ln Q + \frac{1}{2\gamma N} + \frac{1}{N}\int_{i\in\Delta} \ln p_i di + \frac{\gamma}{2N}\int_{i\in\Delta}\int_{j\in\Delta} \ln p_i(\ln p_j - \ln p_i)djdi, \tag{2}$$

where E is the minimum expenditure required to obtain Q, Δ denotes the set of differentiated goods available for purchase, N is the measure of Δ, p_i denotes the price of differentiated good i, and γ indicates the level of substitutability between the varieties (a higher γ implies a higher degree of substitution).[12]

The production of each unit of the homogeneous good requires one unit of labor. This good is sold in a perfectly competitive market at a price of 1 and hence, the wage is 1. Given the quasilinear utility function in equation (1) and the equivalence of the wage and the price of the homogeneous good, the total expenditure in differentiated goods of the representative household is simply given by η, where we must satisfy $\eta < 1$.

The demand of the representative household for differentiated good i is given by $q_i = \sigma_i \frac{\eta}{p_i}$, where σ_i is the share of variety i in the total household expenditure on differentiated goods. By Shephard's lemma—the derivative of equation (2) with respect to $\ln p_i$—we obtain that $\sigma_i = \gamma \ln\left(\frac{\hat{p}}{p_i}\right)$, where

$$\hat{p} = \exp\left(\frac{1}{\gamma N} + \overline{\ln p}\right) \tag{3}$$

denotes the maximum price that firms can set in the differentiated-good sector, and $\overline{\ln p} = \frac{1}{N}\int_{j\in\Delta} \ln p_j dj$.

2.1.2 Pricing and Production of Differentiated Goods

Because households are located in the unit interval, the market demand for differentiated good i is identical to the demand of the representative household. A producer of good i with a constant marginal cost, c_i, who takes $\hat{p}$ as given, sets the price that maximizes $\pi_i = (p_i - c_i)q_i$. This maximization problem yields $p_i = \left[1 + \ln\left(\frac{\hat{p}}{p_i}\right)\right] c_i$, from which we can solve for p_i as

$$p_i = (1 + \mu_i)c_i, \tag{4}$$

where μ_i is producer i's proportional markup over the marginal cost, which is given by

$$\mu_i = \mathbf{\Omega}\left(\frac{\hat{p}}{c_i}e\right) - 1. \tag{5}$$

[12] The translog function in equation (2) implies endogenous markups in the differentiated-good sector. Given the intimate link between competition and firm-level markups, this framework is more appropriate than the typical alternative of using a CES aggregator (which implies exogenous markups).

The function $\mathbf{\Omega}(\cdot)$ denotes the Lambert $\mathcal{W}$ function, which is the inverse of $f(\Omega) = \Omega e^{\Omega}$; that is, in the equation $x = ze^z$, we solve for z as $z = \Omega(x)$. Among its properties, we have that if $x \geq 0$ then $\mathbf{\Omega}'(x) > 0$, $\mathbf{\Omega}''(x) < 0$, $\mathbf{\Omega}(0) = 0$, and $\mathbf{\Omega}(e) = 1$.[13] Note that μ_i is zero if $c_i = \hat{p}$ (so that the price of good i equals its marginal cost), and is greater than zero if $c_i < \hat{p}$. If $c_i > \hat{p}$, firm i will not produce.

Another useful result arising from the properties of the Lambert $\mathcal{W}$ function is that $\ln p_i = \ln \hat{p} - \mu_i$.[14] Using this result in the expression for σ_i in section 2.1.1, yields $\sigma_i = \gamma \mu_i$. That is, the market share density of producer i is directly proportional to its markup.

Firms are heterogeneous in productivity. Following Melitz (2003), I assume that a firm knows its productivity—drawn from a probability distribution—only after paying a sunk entry cost of f_E. Knowing its productivity, the firm can decide between using only domestic labor (L) or use also foreign labor (L^*). The foreign wage, W^*, is less than the domestic wage of 1. I assume that an offshoring firm splits its production process in two complementary parts, one of which stays at home while the other is moved abroad. Let $s \in \{n, o\}$ denote a firm's offshoring status, with n meaning "not offshoring" and o meaning "offshoring". Then, the production function of a producer with productivity φ and offshoring status s is given by $y_s(\varphi) = \varphi \mathbb{L}_s$, where

$$\mathbb{L}_s = \begin{cases} L & \text{if } s = n \\ \min\left\{\frac{L}{1-\alpha}, \frac{L^*}{\alpha}\right\} & \text{if } s = o. \end{cases}$$

In $\mathbb{L}_s$, $\alpha \in (0, 1)$ represents the fraction of the production process being offshored.

Denoting the price of $\mathbb{L}_s$ with W_s, we obtain that $W_n = 1$ and $W_o = 1 - \alpha + \alpha W^*$. Hence, the marginal cost of a firm with productivity φ and offshoring status s is $\frac{W_s}{\varphi}$. Note that $W_o < 1$; therefore, a firm's marginal cost is always lower when offshoring.

Following equations (4) and (5), we write the price set by a firm with productivity φ and offshoring status s as

$$p_s(\varphi) = (1 + \mu_s(\varphi))\frac{W_s}{\varphi},$$

for $s \in \{n, o\}$, and

$$\mu_s(\varphi) = \mathbf{\Omega}\left(\frac{\hat{p}}{\frac{W_s}{\varphi}}e\right) - 1.$$

Then, this firm's equilibrium output and profit functions are respectively given by

$$y_s(\varphi) = \left(\frac{\mu_s(\varphi)}{1 + \mu_s(\varphi)}\right)\frac{\gamma\eta}{\frac{W_s}{\varphi}} \quad \text{and} \quad \pi_s(\varphi) = \frac{\mu_s(\varphi)^2}{1 + \mu_s(\varphi)}\gamma\eta.$$

[13] See Corless *et al.* (1996) for an overview of the Lambert $\mathcal{W}$ function. Other of its properties include $\mathbf{\Omega}'(x) = \frac{\mathbf{\Omega}(x)}{x[1+\mathbf{\Omega}(x)]}$ for $x \neq 0$, and $\ln[\mathbf{\Omega}(x)] = \ln x - \mathbf{\Omega}(x)$ when $x > 0$.

[14] To obtain this result, first we rewrite the price of good i as $p_i = \mathbf{\Omega}\left(\frac{\hat{p}}{c_i}e\right)c_i$, we then take the natural log of that expression and use the property $\ln[\mathbf{\Omega}(x)] = \ln x - \mathbf{\Omega}(x)$ for $x > 0$.

2.1.3 Cutoff Productivity Levels

As a Melitz-type model, cutoff levels determine the tradability of goods: a firm sells its differentiated good if and only if its productivity is no less than the cutoff productivity level for all the firms with the same offshoring status. The existence of the upper bound for the price that firms can set, $\hat{p}$, allows us to obtain the cutoff productivity levels without the need to assume fixed costs of production (which are necessary in the Melitz (2003) model with CES preferences). Using the markup function in the previous section, we define the cutoff productivity level for firms with offshoring status s as

$$\varphi_s = \inf\{\varphi : \mu_s(\varphi) \geq 0\} = \frac{W_s}{\hat{p}}, \tag{6}$$

for $s \in \{n, o\}$. The model's cutoff productivity levels are then φ_n and φ_o.

Note that we can use the zero-cutoff-markup condition in equation (6) to replace $\hat{p}$ in the markup equation from the previous section. Hence, we rewrite the markup of a firm with productivity φ and offshoring status s as

$$\mu_s(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_s}e\right) - 1, \tag{7}$$

for $\varphi \geq \varphi_s$, and $s \in \{n, o\}$. Given the properties of $\mathbf{\Omega}(\cdot)$ from the previous section, $\mu_s(\varphi)$ is strictly increasing in φ; that is, given offshoring status s, more productive firms charge higher markups.

Moreover, combining the two expressions that stem from (6), we obtain one of the two equations we need to solve the model:

$$\varphi_o = W_o \varphi_n, \tag{8}$$

As $W_o < 1$, it is always true that $\varphi_o < \varphi_n$. Hence, a firm whose productivity is in the interval $[\varphi_o, \varphi_n)$ will only produce if it offshores.

2.2 The Offshoring Decision

Following the model of Caballero and Engel (1999) on lumpy investment decisions in a generalized (S, s) framework, I model the offshoring decision on the basis of random adjustment costs. A firm which decides to offshore incurs adjustment costs due to the disruption and/or reorganization of the production process. These costs, however, can vary over time and are not necessarily the same for firms with the same level of productivity.

From section 2.1.2, we know that the total profit obtained every period by a firm with productivity φ and offshoring status s is

$$\pi_s(\varphi) = \begin{cases} 0 & \text{if} \quad \varphi < \varphi_s \\ \frac{\mu_s(\varphi)^2}{1+\mu_s(\varphi)}\gamma\eta & \text{if} \quad \varphi \geq \varphi_s, \end{cases}$$

for $\mu_s(\varphi)$ given by equation (7) and $s \in \{n, o\}$. Since the marginal cost is lower when a firm offshores, it is always the case that $\pi_o(\varphi) \geq \pi_n(\varphi)$, with strict inequality if $\varphi > \varphi_o$. This implies that the offshoring decision is irreversible.

At the beginning of each period, every non-offshoring firm finds out its offshoring adjustment cost, which includes a component that is proportional to the non-offshoring profits plus a component unrelated to the firm's productivity.[15] The firm then decides whether to offshore. If the firm decides to offshore, it will continue offshoring until it is hit by an exogenous death shock. If the firm does not offshore, it can die at the end of the period (after an exogenous death shock), or survive and receive a new adjustment cost at the beginning of the following period. The offshoring adjustment cost for a firm with productivity φ in a certain period is then given by

$$\psi\left(\pi_n(\varphi) + f_o\right),$$

where ψ is random, non-negative, and with cumulative distribution function $F(\psi)$. The term $\psi\pi_n(\varphi)$ accounts for adjustment costs related to the firm's productivity (for given ψ, these costs are increasing in productivity), while ψf_o accounts for adjustment costs that are independent of φ.

As in Melitz (2003), let δ be the probability of an exogenous death shock at the end of each period. In steady state, the per-period profit of an offshoring firm with productivity φ, $\pi_o(\varphi)$, is constant; thus, this firm's expected lifetime profits are $\frac{\pi_o(\varphi)}{\delta}$. Hence, at the beginning of each period, the Bellman equation for the value of a non-offshoring firm with productivity φ and adjustment factor ψ is

$$V(\varphi, \psi) = \max\left\{\frac{\pi_o(\varphi)}{\delta} - \psi\left(\pi_n(\varphi) + f_o\right), \pi_n(\varphi) + (1-\delta)E\left[V(\varphi, \psi')\right]\right\}. \tag{9}$$

Let $\Psi(\varphi)$ be the value for ψ that makes a non-offshoring firm with productivity φ indifferent between offshoring or not. The following proposition describes the solution for $\Psi(\varphi)$.

Proposition 1 ***(The cutoff adjustment factor)***

Given the Bellman equation (9) and a continuous $F(\psi)$, the cutoff adjustment factor for a non-offshoring firm with productivity φ, $\Psi(\varphi)$, is the unique solution to the equation

$$\Psi(\varphi) = \frac{z(\varphi)}{\delta} - \frac{1-\delta}{\delta}\int_0^{\Psi(\varphi)} F(\psi)d\psi, \tag{10}$$

where $z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\pi_n(\varphi) + f_o} \geq 0$ is an adjusted measure of the distance between the firm's offshoring and non-offshoring profits.

Therefore, at the beginning of each period, for the set of non-offshoring firms with productivity φ, those drawing an adjustment factor below $\Psi(\varphi)$ become offshoring firms. We can be more precise and pin down the probability that a non-offshoring firm with productivity φ

[15] Caballero and Engel (1999) interpret adjustment costs that are proportional to the before-change profits as the amount of profits that a firm stops receiving during the adjustment. As mentioned in section 1.1, these costs reflect the fact that the opportunity cost of adjustment is expected to be higher for more profitable firms.

begins to offshore in a particular period. Denoting this probability with $\Lambda(\varphi)$, it follows that $\Lambda(\varphi) = F(\Psi(\varphi))$. The following proposition describes the behavior of $\Lambda(\varphi)$.

Proposition 2 ***(The probability of offshoring)***

1. *$\Lambda(\varphi) = 0$ for $\varphi \leq \varphi_o$, and $\Lambda(\varphi) \to 0$ if $\varphi \to \infty$;*

2. *If $f_o > 0$, there is a unique maximum for $\Lambda(\varphi)$ in the interval (φ_o, ∞). Given φ_n and φ_o, the level of productivity that maximizes $\Lambda(\varphi)$ approaches φ_n from the right as f_o declines;*

3. *If $f_o = 0$, $\Lambda(\varphi) = 1$ for $\varphi \in (\varphi_o, \varphi_n]$, and is strictly decreasing for $\varphi > \varphi_n$.*

Figure 3 presents a graphical description of Proposition 2. The offshoring probability is zero for a firm with productivity at or below φ_o, as this firm cannot make positive profits even if it offshores. For firms with productivities above φ_o, it is useful to refer to the adjusted measure of the incremental profits from offshoring, $z(\varphi)$, which is the most important determinant of the shape of $\Lambda(\varphi)$. The larger is $z(\varphi)$, the higher is the adjustment factor that a non-offshoring firm is willing to accept, and hence the higher the offshoring probability. Non-offshoring firms with productivities between φ_o and φ_n do not produce—have zero profits—and thus, their offshoring decision only depends on the comparison of offshoring profits and the component of adjustment costs unrelated to productivity, ψf_o. If $f_o = 0$, non-offshoring firms in this range face no adjustment costs, and hence all of them become offshoring firms; if $f_o > 0$, these firms' offshoring prospects increase with productivity, and hence $\Lambda(\varphi)$ is increasing in this range. For non-offshoring firms with productivities above φ_n (so that they produce and have positive profits), their offshoring decision also considers the adjustment costs associated with their size, $\psi\pi_n(\varphi)$. For those firms close to φ_n (from the right), they are small enough so that the most important adjustment cost they face is ψf_o. Thus, if $f_o > 0$, there exists a range of firms—starting at φ_n—for which the offshoring probability increases with productivity. As the adjustment cost related to the firm's size becomes more important, there exists a point from which the offshoring probability starts to decline.

There are two key differences of this model compared to heterogeneous-firm models that only consider homogeneous fixed costs of offshoring. In those models, every firm with a productivity no less than a cutoff level will offshore: denoting that cutoff level by $\tilde{\varphi}$, these models imply that $\Lambda(\varphi) = 0$ if $\varphi < \tilde{\varphi}$, and $\Lambda(\varphi) = 1$ if $\varphi \geq \tilde{\varphi}$. On the other hand, in this model *(i)* there is no cutoff level that separates non-offshoring and offshoring firms, and *(ii)* the most productive firms can have offshoring probabilities that are below the offshoring probabilities of much less productive firms.

One of the results that spans from the second part of Proposition 2 is that the range of a positive relationship between productivity and offshoring probability is wider if there is a higher relative importance of f_o. This result gives us an insight into how the offshoring probability function, $\Lambda(\varphi)$, would look like for different industries. In those industries for which offshoring implies large disruptions in the production process—so that the adjustment cost related to profitability is relatively more important—we should expect to see a well-defined inverted-U shape in $\Lambda(\varphi)$. On the other hand, in those industries for which offshoring mostly implies

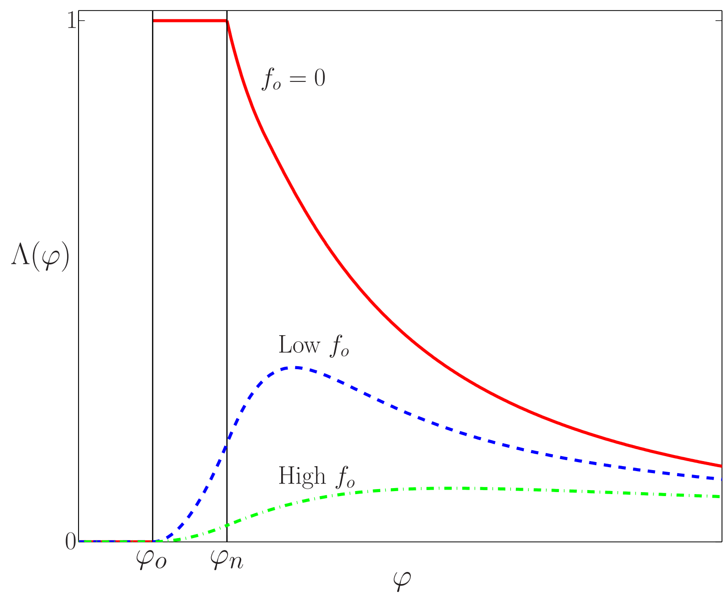


Figure 3: Probability of offshoring

adjustment costs unrelated to firm's productivity, $\Lambda(\varphi)$ will show a weak inverted-U shape and hence, will give the general impression that more productive firms are more likely to offshore.

2.3 Distribution and Composition of Firms

After entry, a firm draws its productivity from the interval $[\varphi_{\min}, \infty)$ according to the cumulative distribution function $G(\varphi)$, with probability density function denoted by $g(\varphi)$. Hence, every period t there is a pool of firms with measure $N_{P,t}$ that contains all the existing firms in the interval $[\varphi_{\min}, \infty)$. Given the exogenous death probability (δ), $N_{P,t+1} = (1-\delta)N_{P,t} + N_{E,t+1}$, where $N_{E,t+1}$ denotes the mass of entrants in $t+1$. In steady state, the measure of the pool of firms is constant at N_P, so that $N_E = \delta N_P$.

The pool of firms, N_P, is composed of offshoring and non-offshoring firms. For each level of productivity, the determinants of the proportions of each type of firm are δ and $\Lambda(\varphi)$. In particular, for productivity level φ, the steady-state proportion of offshoring firms is given by

$$\Gamma(\varphi) = \frac{\Lambda(\varphi)}{\delta + (1-\delta)\Lambda(\varphi)}. \tag{11}$$

Then, the proportion of firms with productivity φ that do not offshore is given by $1 - \Gamma(\varphi)$.[16]

Let $h_o(\varphi)$ and $H_o(\varphi)$ denote, respectively, the probability density function and the cumulative distribution function for the productivity of offshoring firms. Using $\Gamma(\varphi)$ and $g(\varphi)$, I obtain

$$h_o(\varphi) = \frac{\Gamma(\varphi)g(\varphi)}{\bar{\Gamma}}, \tag{12}$$

where $\bar{\Gamma} = \int_{\varphi_{\min}}^{\infty} \Gamma(\varphi)g(\varphi)d\varphi$ is the steady-state proportion of offshoring firms. Hence, the

[16]Note that if the death probability, δ, is equal to zero, all the firms with productivity higher than φ_o offshore in the steady state.

measure of offshoring firms is $\bar{\Gamma}N_P$. Given that $\Gamma(\varphi) = 0$ if $\varphi \leq \varphi_o$, then $\bar{\Gamma} = \int_{\varphi_o}^{\infty} \Gamma(\varphi)g(\varphi)d\varphi$.

Analogously, let $h_n(\varphi)$ and $H_n(\varphi)$ denote the probability density function and the cumulative distribution function for the productivity of non-offshoring firms. We then have that

$$h_n(\varphi) = \frac{[1 - \Gamma(\varphi)]g(\varphi)}{1 - \bar{\Gamma}}. \tag{13}$$

The measure of non-offshoring firms is $(1 - \bar{\Gamma})N_P$.

As mentioned in section 2.1.1, N denotes the measure of the set of goods that are available for purchase. As each firm produces a single good, the set of actual producers also has measure N and is a subset of the pool of firms (which has measure N_P). The set of actual producers comprises non-offshoring firms, with measure N_n, and offshoring firms, with measure N_o. That is, $N = N_n + N_o$.

From the previous sections, we know that a non-offshoring firm produces if its productivity is no less than φ_n. Given that a fraction $1 - H_n(\varphi_n)$ of the pool of non-offshoring firms satisfies that requirement, we get that $N_n = (1 - H_n(\varphi_n))(1 - \bar{\Gamma})N_P$. On the other hand, every offshoring firm has a productivity level that is no less than φ_o—*i.e.* $H_o(\varphi_o) = 0$ and every offshoring firm produces. Thus, $N_o = \bar{\Gamma}N_P$. Adding the expressions for N_n and N_o, we get

$$N = \left[1 - (1 - \bar{\Gamma})H_n(\varphi_n)\right] N_P.$$

The following lemma presents the solution for N_P as a function of the cutoff productivity levels and exogenous parameters.

Lemma 1 ***(The measure of the pool of firms)***

Given the productivity distributions of non-offshoring and offshoring firms, the measure of the pool of firms is given by

$$N_P = \frac{1}{\gamma\left[(1 - \bar{\Gamma})(1 - H_n(\varphi_n))\bar{\mu}_n + \bar{\Gamma}\bar{\mu}_o\right]}, \tag{14}$$

where $\bar{\mu}_s = \int_{\varphi_s}^{\infty} \mu_s(\varphi)h_s(\varphi \mid \varphi \geq \varphi_s)d\varphi$ *is the average markup of producers with offshoring status* s*, for* $s \in \{n, o\}$.

We can also write expressions for average productivities, average prices, and market shares. For producing firms with offshoring status s, for $s \in \{n, o\}$, the average productivity and the average price are respectively given by

$$\bar{\varphi}_s = \int_{\varphi_s}^{\infty} \varphi h_s(\varphi \mid \varphi \geq \varphi_s)d\varphi \quad \text{and} \quad \bar{p}_s = \int_{\varphi_s}^{\infty} p_s(\varphi)h_s(\varphi \mid \varphi \geq \varphi_s)d\varphi.$$

Then, the overall average price can be written as $\bar{p} = \frac{N_n}{N}\bar{p}_n + \frac{N_o}{N}\bar{p}_o$. For market shares, from section 2.1.2 it follows that the market share density of a firm with productivity φ and offshoring status s is $\sigma_s(\varphi) = \gamma\mu_s(\varphi)$: given offshoring status s, more productive firms charge higher markups and have larger market shares. Integrating the previous expression over all firms with the same offshoring status, I obtain that the total market share of firms with offshoring status

s is $\sigma_s = \gamma N_s \bar{\mu}_s$, for $s \in \{n, o\}$, with $\sigma_n + \sigma_o = 1$.

2.4 Free-Entry Condition and Equilibrium

As in Melitz (2003), firms enter as long as the expected value of entry is no less than the sunk entry cost, f_E.

A potential entrant knows that the expected profit of a firm with productivity φ for its first period of existence is

$$\bar{\pi}(\varphi) = (1 - \Lambda(\varphi))\pi_n(\varphi) + \Lambda(\varphi)\left[\pi_o(\varphi) - E(\psi \mid \psi \leq \Psi(\varphi))\left(\pi_n(\varphi) + f_o\right)\right],$$

which is a weighted average between the non-offshoring profits and the offshoring profits minus the expected adjustment cost, with the weights determined by the offshoring probability, $\Lambda(\varphi)$. Taking into account the exogenous death shock at the end of every period, the potential entrant also knows that the expected profit of a firm with productivity φ for its t^{th} period of existence is given by

$$\bar{\pi}^t(\varphi) = (1 - \delta)^{t-1}\left\{(1 - \Lambda(\varphi))^{t-1}\bar{\pi}(\varphi) + \left[1 - (1 - \Lambda(\varphi))^{t-1}\right]\pi_o(\varphi)\right\},$$

where the first term inside the brackets accounts for the expected profit at time t if the firm has not yet decided to offshore by $t - 1$, while the second term accounts for the profit the firm receives at t if it is already offshoring by $t - 1$. Of course, $\bar{\pi}^1(\varphi) = \bar{\pi}(\varphi)$. Hence, given the productivity distribution of new entrants (with probability density function $g(\varphi)$), a potential entrant's expected value of entry is given by $\bar{\pi}_E = \int_{\varphi_o}^{\infty}\left[\sum_{t=1}^{\infty}\bar{\pi}^t(\varphi)\right]g(\varphi)d\varphi$. Substituting the expressions for $\bar{\pi}^t(\varphi)$ and $\bar{\pi}(\varphi)$, $\bar{\pi}_E$ can be written as

$$\bar{\pi}_E = \int_{\varphi_o}^{\infty}\frac{1}{\delta + (1 - \delta)\Lambda(\varphi)}\left\{(1 - \Lambda(\varphi))\pi_n(\varphi) + \Lambda(\varphi)\left[\frac{\pi_o(\varphi)}{\delta} - E(\psi \mid \psi \leq \Psi(\varphi))\left(\pi_n(\varphi) + f_o\right)\right]\right\}g(\varphi)d\varphi. \tag{15}$$

Using equations (11), (12), and (13), I can rewrite equation (15) in terms of $h_o(\varphi)$ and $h_n(\varphi)$. Hence, the free-entry condition, $\bar{\pi}_E = f_E$, is given by

$$(1 - \bar{\Gamma})\int_{\varphi_n}^{\infty}\frac{\pi_n(\varphi)}{\delta}h_n(\varphi)d\varphi + \bar{\Gamma}\int_{\varphi_o}^{\infty}\left[\frac{\pi_o(\varphi)}{\delta} - E(\psi \mid \psi \leq \Psi(\varphi))\left(\pi_n(\varphi) + f_o\right)\right]h_o(\varphi)d\varphi = f_E. \tag{16}$$

Note that the expression for $\bar{\pi}_E$ in the free-entry condition presents the value of entry as a weighted average between a potential entrant's lifetime expected profits if it never offshores, and the expected lifetime offshoring profits minus the one-time adjustment cost, with the weights determined by the steady-state proportion of offshoring firms, $\bar{\Gamma}$.

This concludes the model. After obtaining the functions for $\Psi(\varphi)$ and $\Lambda(\varphi)$, we use equations (8) and (16) to solve for φ_o and φ_n. Once we obtain the cutoff productivity levels, we can solve for the rest of the variables: average markups, average prices, average productivities, market shares, N_P, N_n, N_o, and N.

For the equilibrium to exist and to be unique, the value of entry must decline with the cutoff productivity levels. In particular, after plugging in equation (8) into $\bar{\pi}_E$, we must satisfy

$$\frac{\partial \bar{\pi}_E}{\partial \varphi_o} < 0. \tag{17}$$

In this model, increases in the cutoff levels cause a decrease in $\bar{\pi}_E$ due to declines in expected offshoring and non-offshoring profits, but there is also a counteracting effect in $\bar{\pi}_E$ due to the expected decline in offshoring adjustment costs. For existence and uniqueness, the effect on expected profits must dominate the effect on the expected adjustment cost. A sufficient (but by no means necessary) condition for $\frac{\partial \bar{\pi}_E}{\partial \varphi_o} < 0$ is that $E(\psi) < \frac{1}{\delta}$.

3 The Competitive Environment and the Offshoring Likelihood

This section discusses the model's implications for the effects of changes in the competitive environment on firms' offshoring decisions. I begin by defining what a tougher competitive environment is. Then I identify some of the parameters whose changes create a tougher environment. I continue by discussing the effects of competition on offshoring probability: the Schumpeterian effect and the escape-competition effect. The section concludes with a numerical exercise.

3.1 A Tougher Competitive Environment

This paper takes advantage of the endogenous-markups structure of the model to define in a simple way a tougher competitive environment.

Definition *(A tougher competitive environment)*

A competitive environment is said to be tougher if every producing firm that keeps the same offshoring status is forced to reduce its markup. That is, $\mu_n(\varphi)$ declines for $\varphi \geq \varphi_n$, and $\mu_o(\varphi)$ declines for $\varphi \geq \varphi_o$.

Although we usually relate a tougher environment to firms decreasing their prices, this interpretation is out of reach for traditional CES models, as they imply exogenous markups—and hence fixed firm-level prices. Also, the economic literature tends to associate a tougher environment with an increase in the number of competitors in a market. By our definition above, however, a tougher competitive environment is not necessarily associated with more competitors.

From equation (7) we know that the markup of a firm with offshoring status s and productivity $\varphi \geq \varphi_s$ is given by $\mu_s(\varphi) = \mathbf{\Omega}\left(\frac{\varphi}{\varphi_s}e\right) - 1$, for $s \in \{n, o\}$. From the properties of the Lambert $\mathcal{W}$ function mentioned in section 2.1.2, it follows that this firm's markup declines if and only if φ_s increases. Hence, we write the following corollary to the definition above.

Corollary *(The competitive environment and the cutoff levels)*

A tougher competitive environment occurs if and only if both φ_o and φ_n increase.

But then, what drives a tougher competitive environment? In this paper I focus on three parameters: the parameter of substitutability between varieties, γ; the total expenditure on differentiated goods, η; and the parameter of adjustment costs unrelated to productivity, f_o. Let ζ_γ denote the elasticity of φ_o with respect to γ; *i.e.* $\zeta_\gamma = \frac{d \ln \varphi_o}{d \ln \gamma}$. As $\varphi_o = W_o \varphi_n$ and W_o does not depend on γ, it follows that ζ_γ is also the elasticity of φ_n with respect to γ. Similarly, let ζ_η and ζ_{f_o} denote, respectively, the elasticities of φ_o (and φ_n) with respect to η and f_o. Thus, the following lemma describes how changes in these parameters alter the competitive environment.

Lemma 2 ***(The drivers of a tougher competitive environment)***

If equation (17) holds, so that an equilibrium exists and is unique, a tougher competitive environment occurs if either:

1. *γ increases ($\zeta_\gamma > 0$); or*
2. *η increases ($\zeta_\eta > 0$); or*
3. *f_o declines ($\zeta_{f_o} < 0$).*

An increase in γ creates a tougher competitive environment because goods become less differentiated to the eyes of the consumers. Consider, for example, what has happened recently with the substitutability parameter in the market for desktop and laptop computers: the recent introduction of smartphones and tablets—which include features that years ago were only available in computers—makes computers today look more like each other and therefore, computer manufacturers have to decrease their prices. An increase in η represents an increase in the size of the differentiated-good market. As first shown by Melitz and Ottaviano (2008) in a heterogeneous-firm model with endogenous markups, a larger market induces more entry and hence the environment becomes tougher. Even in this case, the number of sellers in the market does not have to rise—the increase in competition arises from entrants, even if they end up not producing. Lastly, a decline in f_o makes offshoring more attractive and increases the value of entry. As with the increase in η, more entry creates a tougher environment and causes a downward pressure on markups.

3.2 The Schumpeterian and Escape-Competition Effects

The net effect of a change in the competitive environment on a firm's offshoring likelihood is the result of two opposing forces. Following the terminology of Aghion *et al.* (2005), I refer to the negative effect of competition in the offshoring probability as the Schumpeterian effect, and to its positive effect as the escape-competition effect. The magnitude of these effects varies according to each firm's productivity. To understand them better, I analyze carefully the case of an increase in γ.

The offshoring probability for a firm with productivity φ is given by $\Lambda(\varphi) = F(\Psi(\varphi))$, where $\Psi(\varphi)$ is given by the solution to equation (10). Hence, using Leibniz's rule we get that

$$\frac{d\Lambda(\varphi)}{d\gamma} = \left[\frac{f\left(\Psi(\varphi)\right)}{\delta + (1-\delta)\Lambda(\varphi)} \right] \frac{dz(\varphi)}{d\gamma}.$$

Given that the term in brackets is positive, the negative or positive response of $\Lambda(\varphi)$ to a change in γ is entirely determined by the response of $z(\varphi)$. Thus, we can understand the Schumpeterian and escape-competition effects by looking only at $z(\varphi)$. From Proposition 1 we know that

$$z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\pi_n(\varphi) + f_o}. \tag{18}$$

To see both effects in action, let me assume—for the moment—that an increase in γ not only reduces $\mu_n(\varphi)$ and $\mu_o(\varphi)$, but also reduces $\pi_n(\varphi)$ and $\pi_o(\varphi)$.[17] The decline in $\pi_o(\varphi)$ is the only driver of the Schumpeterian effect, and reduces $z(\varphi)$ through the numerator. On the other hand, the decline in $\pi_n(\varphi)$ drives the escape-competition effect through two channels: *(i)* it increases the numerator, which directly counteracts the Schumpeterian effect; and *(ii)* it also reduces the denominator because the offshoring adjustment cost is expected to fall with the decrease in $\pi_n(\varphi)$—similar to the decline in the opportunity cost of disruptions in the model of Holmes, Levine and Schmitz (2012).

The Schumpeterian effect refers to the cleansing effect of competition and affects the offshoring incentives of non-offshoring firms by reducing offshoring profits. On the other hand, the escape-competition effect refers to the impact of increased competition on offshoring incentives, based on the response of a normalized (by offshoring adjustment costs) measure of the incremental profits from offshoring. That is, although a tougher competitive environment may decrease both non-offshoring and offshoring profits, the gap between them—normalized by offshoring adjustment costs—may increase. Intuitively, this effect makes offshoring more attractive as a mean to "escape" competition. A firm's offshoring probability declines if the Schumpeterian effect dominates, and increases if the escape-competition effect dominates.

Above I described a situation in which both effects are present for a non-offshoring firm. However, for some firms only the Schumpeterian effect may be present, and for some others, only the escape-competition effect may exist. To see this, it is useful to show the response of the profit functions to a change in γ. The profit function for a firm with offshoring status s and productivity $\varphi \geq \varphi_s$ is given by $\pi_s(\varphi) = \frac{\mu_s(\varphi)^2}{1+\mu_s(\varphi)}\gamma\eta$. Hence, we obtain

$$\frac{d\pi_s(\varphi)}{d\gamma} = \left[\frac{\mu_s(\varphi) - \zeta_\gamma}{\mu_s(\varphi)}\right]\frac{\pi_s(\varphi)}{\gamma},$$

which is non-negative if $\mu_s(\varphi) \geq \zeta_\gamma$. Thus, even though markups decline with the increase in γ for all producing firms that do not change their offshoring status, profits nonetheless increase for some of these firms. Let $\hat{\varphi}_o$ and $\hat{\varphi}_n$ be the productivity levels such that $\mu_o(\hat{\varphi}_o) = \zeta_\gamma$ and $\mu_n(\hat{\varphi}_n) = \zeta_\gamma$, respectively; $\hat{\varphi}_o < \hat{\varphi}_n$ must hold. Moreover, and to cover all possible cases, I assume that $\varphi_n < \hat{\varphi}_o$. Therefore, after an increase in γ, a non-offshoring firm with productivity φ will fall in one of the following cases: *(i)* $\varphi < \varphi_o$; *(ii)* $\varphi \in [\varphi_o, \varphi_n)$; *(iii)* $\varphi \in [\varphi_n, \hat{\varphi}_o)$; *(iv)* $\varphi \in [\hat{\varphi}_o, \hat{\varphi}_n)$; and *(v)* $\varphi \geq \hat{\varphi}_n$.

In the first case, the firm does not make positive profits even if it offshores and hence, none of the effects is present. In the second case, the non-offshoring firm does not produce—it begins to produce if and only if it becomes an offshoring firm—and therefore, only the Schumpeterian effect is present ($\pi_o(\varphi)$ decreases with γ). In the third case, the non-offshoring firm produces

[17] Deviations from the "for the moment" assumption will be explained below.

and both effects are present; both $\pi_n(\varphi)$ and $\pi_o(\varphi)$ decrease with γ. In the fourth case, $\pi_o(\varphi)$ increases with γ and hence there is no Schumpeterian effect; moreover, $\pi_n(\varphi)$ continues to decline and thus, there is a reinforced escape-competition effect. Lastly, in the fifth case, both $\pi_n(\varphi)$ and $\pi_o(\varphi)$ increase with γ; there is a weak escape-competition effect, as the increase in $\pi_n(\varphi)$ has a negative effect in the firm's incentives to offshore (we show below that the negative effect never dominates the positive effect of $\pi_o(\varphi)$).

But how do explain increases in profits in spite of declines in markups? From $\pi_s(\varphi)$ above, note that γ also directly increases profits. This is because the substitutability parameter magnifies the market shares of firms with lower prices. Indeed, at the end of section 2.3 we obtained that the market share density of a firm with productivity φ and offshoring status s is given by $\sigma_s(\varphi) = \gamma\mu_s(\varphi)$. We can then obtain that $\frac{d\ln\sigma_s(\varphi)}{d\ln\gamma} = 1 - \left[\frac{1+\mu_s(\varphi)}{2+\mu_s(\varphi)}\right]\frac{\zeta_\gamma}{\mu_s(\varphi)}$. Given that the term in brackets is always less than 1, note that $\sigma_s(\varphi)$ increases with an increase in γ not only if $\mu_s(\varphi) \geq \zeta_\gamma$, but also for some firms with markups smaller than ζ_γ. Hence, the decrease in markups due to an increase in the substitutability parameter allows a range of firms to capture larger market shares, and a subset of these firms are even able to increase their profits.

The case of an increase in η is very similar to an increase in γ. We also obtain cases of declining markups and increasing profits. However, for the case of η the market shares decline. Profits increase in spite of declining market shares because the market itself is larger; that is, a range of firms are able to sell to more output than before, and profits increase for a subset of these firms.

For the case of a decline in f_o, note first from equation (18) that there is a direct positive effect on $z(\varphi)$—f_o is in the denominator of $z(\varphi)$. Hence, in contrast to an increase in γ, the escape-competition effect is present for every firm with productivity no less than φ_o (even if $\pi_n(\varphi) = 0$). Also, we obtain that for a firm with offshoring status s and productivity $\varphi \geq \varphi_s$, both the markup and profits fall; that is, for $\varphi \geq \varphi_s$, not only $\frac{d\mu_s(\varphi)}{df_o} > 0$ but also $\frac{d\pi_s(\varphi)}{df_o} > 0$. Therefore, for a decline in f_o, a non-offshoring firm with productivity φ may be in one of the following three cases: *(i)* $\varphi < \varphi_o$ (no effects are present); *(ii)* $\varphi \in [\varphi_o, \varphi_n)$ (both effects are present, with the escape-competition effect driven by the direct effect of f_o on $z(\varphi)$); and *(iii)* $\varphi \geq \varphi_n$ (both effects are present, with the escape-competition effect driven by both the direct effect of f_o and the indirect effect through the decline in $\pi_n(\varphi)$).

To conclude this section, it is important to note that for a shock that creates a tougher competitive environment, there exists a cutoff level that separates firms according to the dominating effect. The following proposition describes this result.

Proposition 3 ***(The offshoring probability in a tougher competitive environment)***
If $f_o > 0$, and for a tougher competitive environment driven either by an increase in γ or η, or by a decline in f_o:

1. *A unique productivity level, φ^*, separates non-offshoring firms according to the dominant competition effect.*

2. *For a non-offshoring firm with productivity φ, the Schumpeterian effect dominates if $\varphi < \varphi^*$, while the escape-competition effect dominates if $\varphi > \varphi^*$. Hence, the offshoring*

probability declines if $\varphi < \varphi^*$, and increases if $\varphi > \varphi^*$.

This result differs drastically from the implications of a model with homogeneous fixed costs of offshoring. In such a model, a tougher competitive environment shifts up the offshoring cutoff productivity level—recall that in these models there is a cutoff productivity level that separates non-offshoring and offshoring firms. Hence, we would only obtain a simplified version of the Schumpeterian effect: the probability of offshoring drops from 1 to 0 for the firms between the old and new cutoff level.

3.3 Numerical Exercise

This section presents a numerical exercise that summarizes the model. I begin by specifying the values for the parameters, and the probability distributions for firm-level productivity, $G(\varphi)$, and the offshoring adjustment factor, $F(\psi)$. Then I present the solution for the benchmark case, and lastly I show the model's responses to changes in the competitive environment.

3.3.1 Benchmark Case

Based on the findings of Combes *et al.* (2012), I assume that firm productivity is log-normally distributed with parameters m and v; that is, the probability density function, $g(\varphi)$, is given by

$$g(\varphi) = \frac{1}{\varphi\sqrt{2\pi v}} e^{-\frac{(\ln\varphi - m)^2}{2v}},$$

and $G(\varphi)$ is the corresponding cumulative distribution function. Using the estimated values of Combes *et al.* (2012) for French firms, I set m to -0.05, and v to 0.32. Following Bergin and Feenstra (2009), I set the parameter of substitutability among varieties, γ, at 1. As mentioned in section 2.1.1, the domestic wage is 1. The parameter of preference for differentiated goods, η, is set at 0.5. As η is equivalent to the total expenditure in differentiated goods, the assumed value implies that the representative household spends 50% of its income on differentiated goods. I assume $\alpha = 0.5$, which means that every offshoring firm ships abroad half of its production process. The wage in the foreign country, W^*, is set at 0.5. The values for α and W^* imply that $W_o = 0.75$—the marginal cost for each firm is 25% lower if it offshores. Regarding the offshoring adjustment costs, I assume that the adjustment factor, ψ, follows an uniform distribution in the interval $[0, \psi_{\max}]$; that is, $F(\psi) = \frac{\psi}{\psi_{\max}}$. Based on the finding of Bernard *et al.* (2007) about the proportion of U.S. manufacturing firms involved in importing activities, I set $\psi_{\max}$ and f_o so that about 14% of producing firms offshore. Thus, I set $\psi_{\max} = 100$ and $f_o = 0.2$. The value of the sunk entry cost, f_E, is set at 1.5. Lastly, consistent with U.S. evidence on annual job destruction (see Davis, Faberman and Haltiwanger, 2006), I set the death rate, δ, at 0.1.

The first column of Table 1 presents the solution for the benchmark case. The other columns in Table 1 show the model's responses to a tougher competitive environment. This section discusses the benchmark results, and section 3.3.2 discusses the rest of the results.

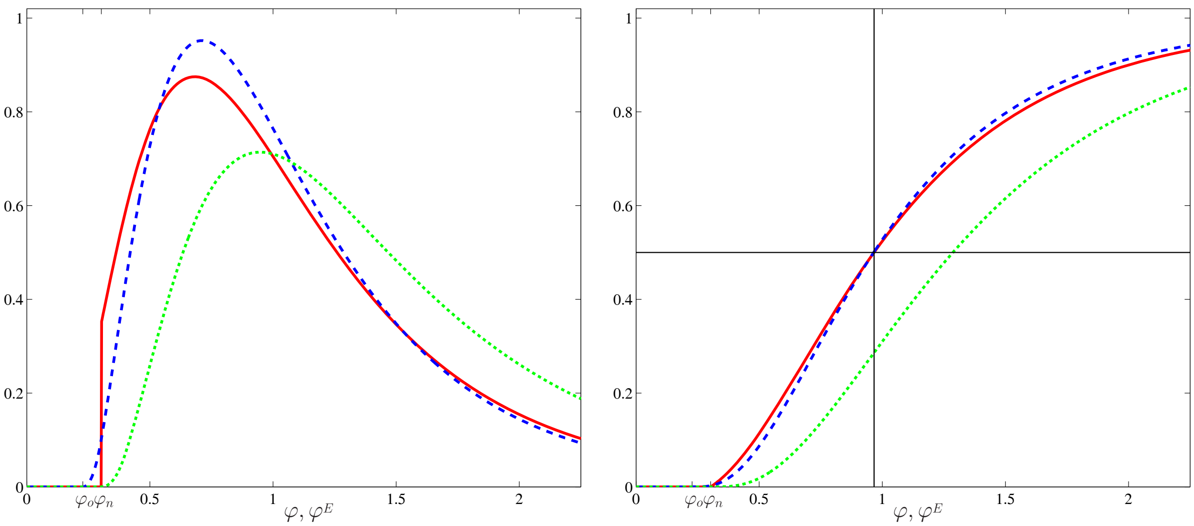
Note that the average productivity levels follow the same order as the cutoff levels: $\varphi_o < \varphi_n$ and $\bar{\varphi}_o < \bar{\varphi}_n$. Hence, contrary to a heterogeneous-firm model with homogeneous fixed costs

Table 1: Model's Numerical Exercise

	Benchmark	Increase in γ (to 2)	Increase in η (to 0.75)	Decline in f_o (to 0.05)
Productivity:				
φ_o	0.228	0.369	0.307	0.231
φ_n	0.303	0.492	0.410	0.308
$\bar{\varphi}_o$	1.125	1.203	1.164	1.028
$\bar{\varphi}_n$	1.141	1.225	1.180	1.173
$\bar{\varphi}_o^E$	1.500	1.604	1.552	1.370
Prices:				
$\hat{p}$	3.297	2.032	2.442	3.246
$\bar{p}$	1.729	1.314	1.472	1.687
$\bar{p}_o$	1.469	1.142	1.265	1.551
$\bar{p}_n$	1.771	1.356	1.515	1.729
Markups and shares:				
$\bar{\mu}_o$	0.860	0.612	0.700	0.790
$\bar{\mu}_n$	0.677	0.442	0.522	0.685
σ_o	0.170	0.251	0.215	0.261
σ_n	0.830	0.749	0.785	0.739
Composition of firms:				
N	1.424	1.052	1.810	1.409
N_o	0.198	0.205	0.307	0.331
N_n	1.226	0.847	1.503	1.078
$\frac{N_o}{N}$	0.139	0.195	0.170	0.235
Dominant effect cutoff:				
φ^*	—	0.561	0.511	0.235

of offshoring, in this model the average productivity of offshoring firms can be lower than the average productivity of non-offshoring firms. This result can be reversed (*i.e.* obtain $\varphi_o < \varphi_n$ and $\bar{\varphi}_o > \bar{\varphi}_n$) for larger levels of f_o, as the offshoring adjustment cost unrelated to the firm's productivity level, ψf_o, is the main driver in the offshoring decisions of low productivity firms.

There is another important point to make with respect to average productivities. The general finding in the empirical literature is that firms engaging in international activities are on average more productive than purely-domestic firms. In the previous paragraph we compared the average initial (or ex-ante) productivities for offshoring and non-offshoring firms and obtained the opposite result; however, for offshoring firms, $\bar{\varphi}_o$ does not represent the average *effective* (or ex-post) productivity. As mentioned in the Introduction, the average effective productivity of offshoring firms incorporates the decline in marginal costs due to offshoring—the so-called productivity effect of offshoring. Hence, denoting by φ^E the effective productivity of an offshoring firm with initial productivity φ, it follows that $\varphi^E = \frac{\varphi}{W_o}$ (this is the inverse of the offshoring firm's marginal cost—see section 2.1.2). Therefore, using $\bar{\varphi}_o^E$ to denote the average effective productivity of offshoring firms, it follows that $\bar{\varphi}_o^E = \frac{\bar{\varphi}_o}{W_o}$. Of course, the average effective productivity of non-offshoring firms continues to be $\bar{\varphi}_n$. Comparing average effective productivities,



(a) Probability density functions

(b) Cumulative distribution functions

Figure 4: Productivity distributions: producing non-offshoring firms (*solid*) and offshoring firms (ex-ante (*dashed*) and effective (*dotted*))

we can see that—as in the empirical evidence—the average effective productivity of offshoring firms is higher than the average effective productivity of non-offshoring firms. Hence, in this example—and in contrast to the long-held self-selection view—offshoring firms are on average (effectively) more productive than non-offshoring firms precisely because they offshore, not the other way around.

To shed more light on this result, let us now look at the ex-ante and effective productivity distributions for offshoring firms, along with the productivity distribution for producing non-offshoring firms. To obtain the probability density function for the effective productivity of offshoring firms, $h_o^E(\varphi^E)$, we only need to apply a change of variable on the density function for the ex-ante productivity of offshoring firms, $h_o(\varphi)$, obtained in equation (12). Given that $\varphi^E = \frac{\varphi}{W_o}$, it follows that $h_o^E(\varphi^E) = W_o h_o(W_o \varphi^E)$. While every offshoring firm produces ($\varphi \geq \varphi_o$ for every offshoring firm), there are some non-offshoring firms whose productivities are below φ_n and hence do not produce; thus, the density function for the productivity of producing non-offshoring firms is given by $h_n(\varphi \mid \varphi \geq \varphi_n)$.[18] Figure 4a shows $h_o(\varphi)$, $h_o^E(\varphi^E)$, and $h_n(\varphi \mid \varphi \geq \varphi_n)$, and Figure 4b presents the corresponding cumulative distribution functions: $H_o(\varphi)$, $H_o^E(\varphi^E)$, and $H_n(\varphi \mid \varphi \geq \varphi_n)$. Note that $h_o(\varphi)$ and $h_n(\varphi \mid \varphi \geq \varphi_n)$ are very close to each other, they have almost the same median, and—as shown in Table 1—$\bar{\varphi}_n$ is only 1.4% higher than $\bar{\varphi}_o$. On the other hand, the offshoring productivity effect shifts to the right and dilates the productivity distribution of offshoring firms, creating a 31.5% gap between the average effective productivity of offshoring firms, $\bar{\varphi}_o^E$, and the average productivity of producing non-offshoring firms, $\bar{\varphi}_n$. The dilation observed in $h_o^E(\varphi^E)$ when compared to $h_o(\varphi)$ occurs because this model's offshoring productivity effect is larger (in absolute terms) for ex-ante more productive firms.[19]

[18] That is, for offshoring firms $H_o(\varphi_o) = 0$ and hence $h_o(\varphi \mid \varphi \geq \varphi_o) = h_o(\varphi)$ for every φ, while for non-offshoring firms $H_n(\varphi_n) > 0$ and hence $h_n(\varphi \mid \varphi \geq \varphi_n) \neq h_n(\varphi)$ for every φ.

[19] For example, with $W_o = 0.75$, the productivity jumps to 0.53 for a new offshoring with an ex-ante productivity

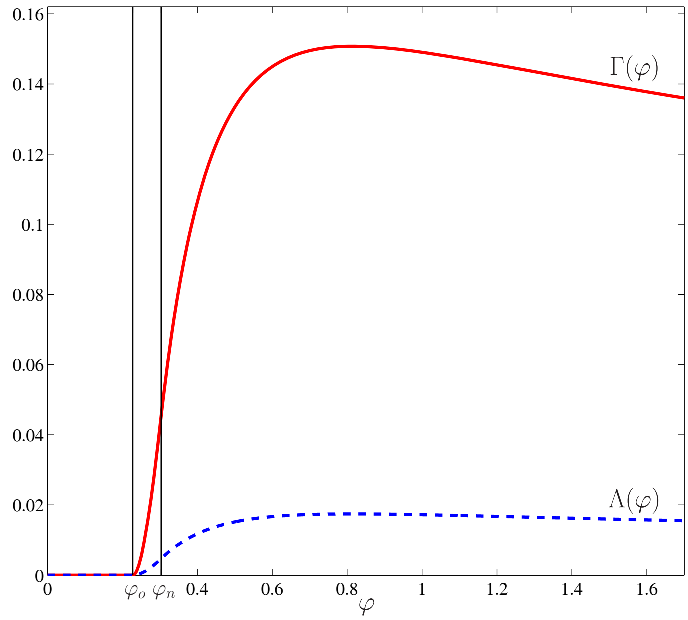


Figure 5: Offshoring probability and proportion of offshoring firms

Importantly, the productivity distributions implied by my model differ drastically from the distributions described in Figure 1 for a homogeneous-fixed-cost model in which only the most productive firms offshore. Moreover, the model generates bell-shaped density functions and cumulative distribution functions that share strong similarities with the empirical distributions described in Figure 2—from Tomiura (2007)—for Japan's offshoring and purely-domestic firms.

Let us now look at the rest of the benchmark results in Table 1. For prices, recall that $\hat{p} = 3.297$ is the maximum price that firms can charge in the differentiated-good market. The average price of offshoring firms, $\bar{p}_o$, is smaller than the average price of non-offshoring firms, $\bar{p}_n$. Nevertheless, even with lower average prices, offshoring firms obtain higher average markups than non-offshoring firms: $\bar{\mu}_o > \bar{\mu}_n$. For the masses of firms, note that although offshoring firms (N_o) represent about 13.9% of the total mass of producing firms (N), their market share (σ_o) is 17%. That is, offshoring firms capture a larger part of the market through their lower prices, limiting the number of competitors.

Lastly, Figure 5 shows the offshoring probability for non-offshoring firms, $\Lambda(\varphi)$, along with the proportion of offshoring firms for each level of (ex-ante) productivity, $\Gamma(\varphi)$. Note that $\Gamma(\varphi)$ looks like a scaled function of $\Lambda(\varphi)$. Indeed, taking the derivative of equation (11), we get that $\text{sgn}(\Gamma'(\varphi)) = \text{sgn}(\Lambda'(\varphi))$. Non-offshoring firms with a productivity of 0.81 have the greatest offshoring probability each period (about 1.74%), and among all the firms with that level of productivity, about 15.1% are offshoring. Note that in this example, the non-offshoring firms with the highest offshoring incentives have a productivity level well below $\bar{\varphi}_n$.

level of 0.4, while it jumps to 2.66 for an offshoring firm with an ex-ante productivity level of 2. The study of Combes *et al.* (2012) on selection and agglomeration effects of large cities provides an illustrative guide for the interpretation of truncation, right-shifting, and dilation of firm-level productivity distributions.

3.3.2 Changes in the Competitive Environment

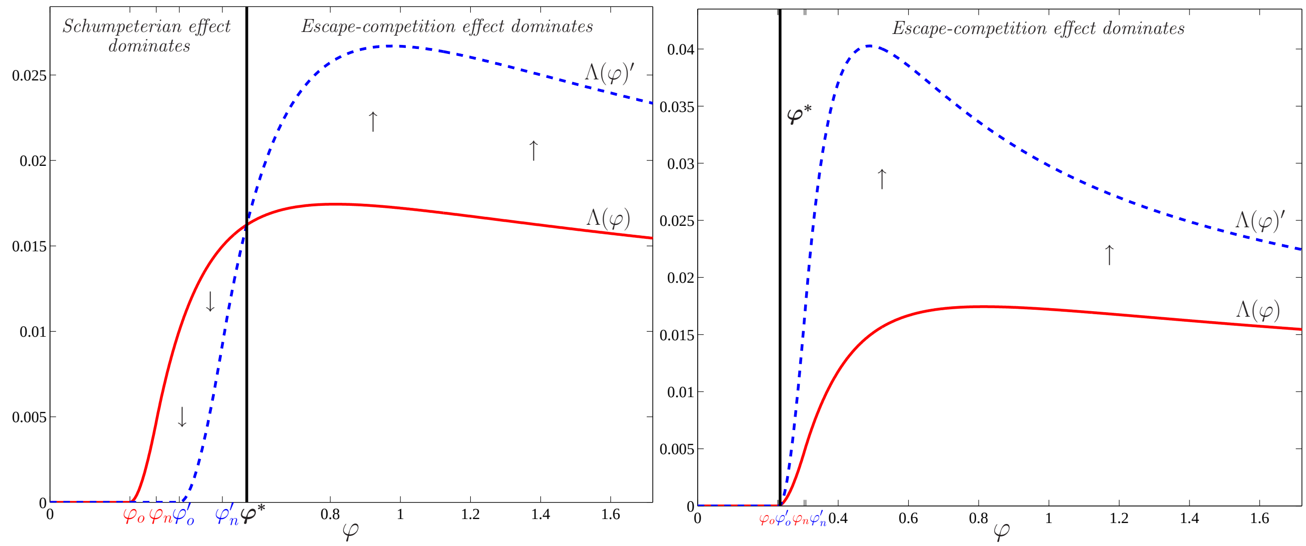
For the numerical example, this section discusses the model's responses to a tougher competitive environment. Following Lemma 2, I consider an increase in the substitutability parameter, γ, from 1 to 2, an increase in market size, η, from 0.5 to 0.75, and a decline in the offshoring adjustment costs unrelated to profitability, f_o, from 0.2 to 0.05. The second, third, and fourth columns in Table 1 present the new steady-state solutions.

From Lemma 2 we know that any of these changes increases both cutoff productivity levels, φ_o and φ_n, which then implies lower markups for producing firms that do not change their offshoring status (satisfying the definition of "tougher competitive environment"). For increases in γ and η, average productivities $\bar{\varphi}_o$, $\bar{\varphi}_n$, and $\bar{\varphi}_o^E$ also increase and the ordering of the benchmark case prevails: $\bar{\varphi}_o < \bar{\varphi}_n < \bar{\varphi}_o^E$. In those two cases, the average productivity ratios $\frac{\bar{\varphi}_n}{\bar{\varphi}_o}$ and $\frac{\bar{\varphi}_o^E}{\bar{\varphi}_n}$ remain very close to their benchmark levels. For a decline in f_o, however, $\bar{\varphi}_o$ and $\bar{\varphi}_o^E$ decline, while $\bar{\varphi}_n$ increases. This is the case because ψf_o is the main driver of offshoring decisions for low-productivity firms (and the only driver for those firms between φ_o and φ_n); therefore, the decline in f_o causes a large increase in the number of low-productivity firms that begin to offshore, causing a decline in the average productivity of offshoring firms. Though the ordering of the benchmark case prevails, $\bar{\varphi}_n$ is now 14.2% higher than $\bar{\varphi}_o$, and $\bar{\varphi}_o^E$ is now only 16.8% higher than $\bar{\varphi}_n$.

The maximum price that firms can set, $\hat{p}$, the overall average price, $\bar{p}$, and the average price of non-offshoring firms, $\bar{p}_n$, decline in all cases. On the other hand, the average price of offshoring firms, $\bar{p}_o$, falls for an increase in either γ or η, but increases for the decline in f_o. The last result does not mean that offshoring firms are increasing their prices after a decline in f_o, but is a consequence of the change in the composition of offshoring firms towards less productive firms. In all cases, $\bar{p}_o$ remains below $\bar{p}_n$.

In all cases, the average markup of offshoring firms, $\bar{\mu}_o$, declines. The average markup of non-offshoring firms, $\bar{\mu}_n$, declines for the increases in γ and η, but increases for the decline in f_o. For the last case, the increase in $\bar{\mu}_n$ in a tougher competitive environment may seem puzzling given that *(i)* the markup of each surviving firm that does not change its offshoring status declines, and *(ii)* the average price of non-offshoring firms also declines. As with the increase in $\bar{p}_o$, the increase in $\bar{\mu}_n$ is the result of a composition effect that creates a strong bias. In particular, the new $\bar{\mu}_n$ no longer includes low-productivity firms that stop producing (those non-offshoring firms with productivities between the old and new φ_n) and firms that begin to offshore. Therefore, this model suggests that in the presence of composition effects, average markups may not be good indicators of the level of competition in a market.

The total mass of producing firms, N, declines after the increase in γ and after the decline in f_o; hence, a tougher competitive environment is not necessarily associated with a larger mass of competitors. For the components of N, note that the mass of offshoring firms, N_o, increases in all cases, while the mass of non-offshoring firms, N_n, declines for the changes in γ and f_o but increases after the increase in η. In the end, the proportion of offshoring firms (along with their market share, σ_o) increases in all cases. In other exercises we can also obtain more concentration even when the market size is bigger (*i.e.* a decline in N after an increase in η), as new and



(a) After an increase in γ

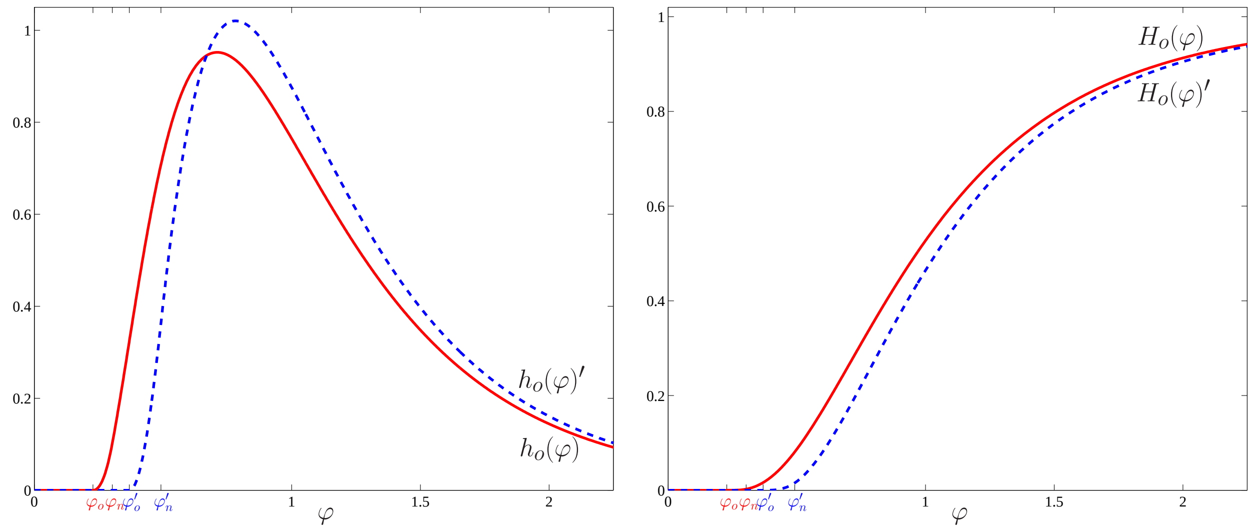
(b) After a decline in f_o

Figure 6: Changes in the offshoring probability

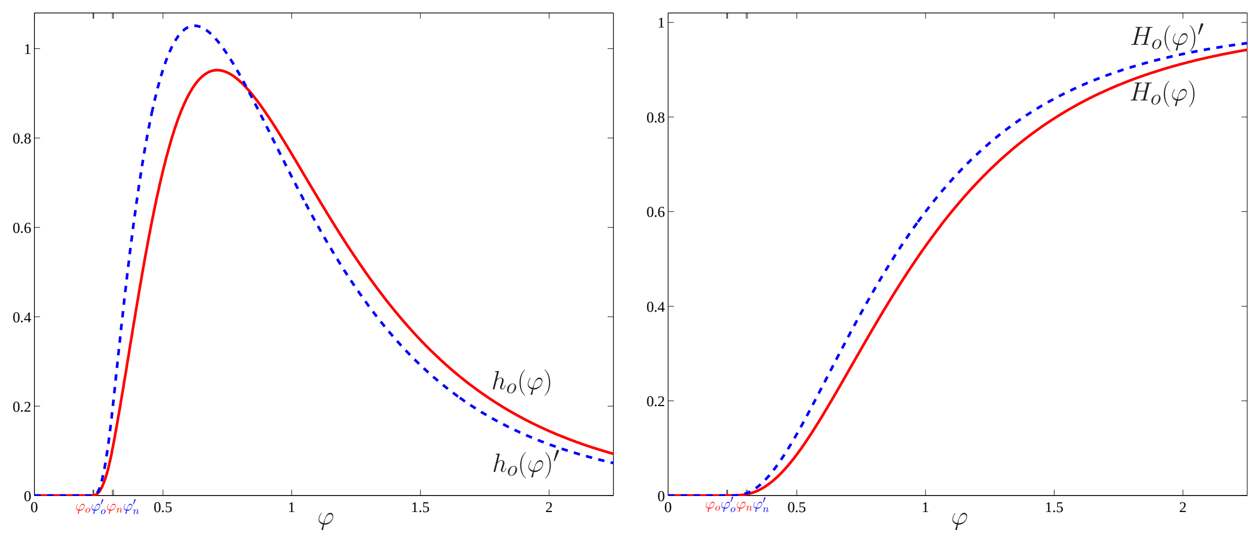
existing offshoring firms replace and steal market share away from non-offshoring firms.

For non-offshoring firms, let us now look into the effects of a tougher competitive environment on the probability of offshoring, $\Lambda(\varphi)$. For each of the three shocks considered in this numerical example, the last row in Table 1 shows φ^*, which is the productivity level that separates non-offshoring firms according to the dominant competition effect. From Proposition 3, we know that the offshoring probability declines for firms to the left of φ^*—where the Schumpeterian effect dominates—and increases for firms to the right of φ^*—where the escape-competition effect dominates. Note that for the increases in γ and η, the values for φ^* in Table 1 are greater than the new steady-state values for φ_n. On the other hand, for the decline in f_o, φ^* is between the new φ_o and the benchmark steady-state level for φ_n. Hence, the range of φ for which the escape-competition effect dominates is wider in the case of the decline in f_o. As a graphical description of Proposition 3, Figure 6 presents the offshoring probability functions in the benchmark steady state, $\Lambda(\varphi)$, and in the new steady state, $\Lambda(\varphi)'$, for the cases of the increase in γ and the decline in f_o. I omit in the figure the case of the increase in η because the plot is very similar to the plot for the increase in γ. The offshoring probability jumps down to zero for each firm between the old and new φ_o. Also, note that the range of φ for which the Schumpeterian effect dominates in Figure 6b is hardly noticeable, as opposed to the equivalent range in Figure 6a. Intuitively, and as seen in section 3.2, this difference arises because for the case of an increase in γ, the Schumpeterian effect is the only effect present for firms with productivities below φ_n, while the escape-competition effect is present for the case of a decline in f_o as long as the productivity of a firm is no less than φ_o.

Lastly, Figure 7 shows the effects of the increase in γ and the decline in f_o on the ex-ante productivity distribution of offshoring firms (similar changes happen in the effective distribution of productivity). As in the previous figure, the plots corresponding to the increase in η are very similar to those obtained for the increase in γ. For each case, the plots show the density functions



(a) Increase in γ: probability density function (left) and cumulative distribution function (right)



(b) Decline in f_o: probability density function (left) and cumulative distribution function (right)

Figure 7: Changes in the distribution of offshoring firms

and the cumulative distribution functions for the benchmark steady state ($h_o(\varphi)$ and $H_o(\varphi)$) and for the new steady state ($h_o(\varphi)'$ and $H_o(\varphi)'$). For the increase in γ, Figure 7a shows that the distribution of offshoring firms shifts to the right and hence, the average productivity of offshoring firms increases. Meanwhile, after the decline in f_o, the distribution of offshoring firms becomes more concentrated in lower levels of productivity, causing a decline in their average productivity.

4 A Technology-Upgrading Application

The framework in this paper has a natural application to the study of technology-upgrading decisions. This section briefly describes how the model above can be rewritten as a technology-upgrading model. In this context, I then review some highlights of the literature on trade and technology upgrading.

4.1 The Technology-Upgrading Model

The model above only needs minor modifications in sections 2.1.2 and 2.1.3. In particular, I assume that a firm's productivity is determined by two components: a technology component, which is common to many firms, and an idiosyncratic component, which can be thought of as the firm's intrinsic *ability*. Every firm is born with the most basic technology, that I label *normal technology*, and with a level of ability that is drawn from a probability distribution. The production function of a firm with normal technology and ability φ is then given by $y_n(\varphi) = A_n \varphi L$, where A_n denotes the firm's normal-technology factor and L represents labor—the only factor of production.

Knowing its ability, the firm can decide whether to upgrade to a better technology, labeled as *outstanding technology*. The production function of a firm with outstanding technology is $y_o(\varphi) = A_o \varphi L$, with $A_o > A_n$. Assuming that the wage is equal to 1, it follows that the marginal cost of a firm with ability φ and technology s is $\frac{1}{A_s \varphi}$, for $s \in \{n, o\}$. Therefore, a firm's marginal cost is always lower when it upgrades its technology. For the price, markup, output, and profit equations for a firm with ability φ and technology s, we only need to replace W_s with $\frac{1}{A_s}$ in the expressions in section 2.1.2.

The cutoff ability level for firms with technology s is then

$$\varphi_s = \inf\{\varphi : \mu_s(\varphi) \geq 0\} = \frac{1}{A_s \hat{p}}$$

for $s \in \{n, o\}$. Thus, the markup function in equation (7) remains unaltered, but the relationship between φ_o and φ_n is now given by

$$\varphi_o = \frac{A_n}{A_o} \varphi_n,$$

As $\frac{A_n}{A_o} < 1$, it follows that $\varphi_o < \varphi_n$.

The rest of the model proceeds as before and hence, we reach the same results: *(i)* low-ability firms with outstanding technology coexist with high-ability firms with normal technology; *(ii)* there is an inverted-U relationship between ability and technology-upgrading likelihood; and *(iii)* after a shock that creates a tougher competitive environment, the interaction between the Schumpeterian and escape-competition effects determines whether the technology-upgrading probability for a normal-technology firm decreases or increases—an ability level, φ^*, separates normal-technology firms according to the dominant effect, with the Schumpeterian effect dominating for the least able firms.

4.2 Discussion

The technology-upgrading model can be further extended to a multi-country setting to study the effects of trade liberalization on firms' technology-upgrading decisions. Hence, it is useful to review and discuss some highlights from the trade and technology-upgrading literature.

First, it is important to note that the literature on competition and innovation mentioned in section 1.1 also has strong links with the literature on trade and technology-upgrading decisions. Indeed, substantial empirical evidence surveyed by Holmes, Levine and Schmitz (2012) and Syverson (2011) on the effects of competition on technology adoption is based on trade liberalization studies. Among these studies, for example, Bloom, Draca and Van Reenen (2011) find that increased import competition from China induces within-firm technical change in European countries.

There are also models with heterogeneous firms that base technology-upgrading decisions on homogeneous fixed costs. For example, Bustos (2011) proposes a model with homogeneous fixed costs of exporting and technology upgrading, and sorts firms according to their capacity to cover these fixed costs.[20] Although her model predicts that only the most productive firms use the best technology, her empirical results—for Argentinian firms facing tariff reductions from Brazil—suggest an inverted-U relationship between firm size and technology-upgrading likelihood. If the largest Argentinian firms face higher technology-upgrading adjustment costs, then the model in this paper provides an appealing explanation to Bustos's findings.

Other theoretical papers highlighting within-firm productivity growth in heterogeneous-firm settings include Costantini and Melitz (2008), Atkeson and Burstein (2010), and Burstein and Melitz (2011). With more sophisticated process-innovation structures, these models also find that exporters—who are already the most productive firms—are also more likely to innovate. If most productive firms are also more likely to upgrade their technologies, then a polarization between firms must occur: the least productive firms remain low-productive through time, whereas the most productive firms become even more productive. Therefore, these models have strong implications for the evolution of the distribution of firm-level productivity. The framework in this paper can be used to understand possible sources of discrepancies between these models' implied distributions and empirical productivity distributions.

5 Conclusion

Recent models of offshoring with heterogeneous firms rely on homogenous fixed costs to characterize firms' offshoring decisions. However, as a decision that implies a reorganization of a firm's production process, it is unlikely that firms with different sizes and levels of productivity face the same offshoring costs. In the spirit of innovation and investment models, this paper proposes a new framework to model offshoring decisions. Based on random adjustment costs of offshoring, the model derives an inverted-U relationship between firm-level productivity and offshoring likelihood. In this context, low-productivity offshoring firms coexist with high-productivity non-offshoring firms.

The model finds results that are out of reach of standard offshoring models. In standard models, a tougher competitive environment makes offshoring more difficult, as less firms are able

[20]The model of Bustos (2011) follows a similar approach to the offshoring model of Antràs and Helpman (2004) described in footnote 2. Assuming a difference between exporting and technology-upgrading fixed costs, Bustos sorts producing firms into three groups: the most productive firms export and use the best technology, the least productive producing firms only sell for the domestic market and use the regular technology, and firms in the middle export and use the regular technology.

to cover the homogeneous fixed cost of offshoring—only a Schumpeterian effect is present. On the other hand, this model also obtains an escape-competition effect by which the offshoring incentives may increase, even if offshoring and non-offshoring profits decline with the tougher environment.

The model derives other suggestive results. For example, though the long-held view is that a high productivity is necessary to offshore, this model's numerical example shows that the higher average productivity of offshoring firms may only be due to the firms' productivity gains driven by offshoring decisions—removing the productivity effect of offshoring, the average productivity of offshoring firms can be below the average productivity of non-offshoring firms. The model also highlights the existence of composition effects in average markups and average prices, which make them subject to sample-selection bias.

Besides the technology-upgrading application considered in section 4, the model can be easily extended to a multi-country framework to answer questions about the effects of trade liberalization on offshoring or innovation decisions. The model can also accommodate more sophisticated offshoring structures—as in Grossman and Rossi-Hansberg (2008)—to study the effects of competition and trade liberalization on employment and wages of different skill groups. Likewise, this framework can be used in models of production organization and trade. In particular, we can use different specifications of the offshoring adjustment cost function to account for different forms of offshoring.

A Appendix: Proofs

Proof of Proposition 1. The variable $\Psi(\varphi)$ denotes the value for ψ that makes a firm with productivity φ indifferent between innovation or not. In the Bellman equation (9), this implies that

$$\frac{\pi_o(\varphi)}{\delta} - \Psi(\varphi)\left(\pi_n(\varphi) + f_o\right) = \pi_n(\varphi) + (1-\delta)E\left[V(\varphi, \psi')\right].$$

Solving for $\Psi(\varphi)$ we obtain

$$\Psi(\varphi) = \frac{1}{\pi_n(\varphi) + f_o}\left(\frac{\pi_o(\varphi)}{\delta} - \pi_n(\varphi)\right) - \frac{1-\delta}{\pi_n(\varphi) + f_o}E\left[V(\varphi, \psi')\right]. \tag{A-1}$$

Given $\Psi(\varphi)$, we can rewrite the value function as

$$V(\varphi, \psi) = \begin{cases} \frac{\pi_o(\varphi)}{\delta} - \psi\left(\pi_n(\varphi) + f_o\right) & \text{if } \psi \leq \Psi(\varphi) \\ \frac{\pi_o(\varphi)}{\delta} - \Psi(\varphi)\left(\pi_n(\varphi) + f_o\right) & \text{if } \psi > \Psi(\varphi). \end{cases}$$

From this expression, we can then get that

$$E\left[V(\varphi, \psi')\right] = \frac{\pi_o(\varphi)}{\delta} - E\left[\min\left\{\psi', \Psi(\varphi)\right\}\right]\left(\pi_n(\varphi) + f_o\right). \tag{A-2}$$

Plugging in equation (A-2) into equation (A-1), we find that

$$\Psi(\varphi) = z(\varphi) + (1-\delta)E\left[\min\left\{\psi', \Psi(\varphi)\right\}\right], \tag{A-3}$$

where $z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\pi_n(\varphi) + f_o}$. Note that $z(\varphi) \geq 0$, as $\pi_o(\varphi) \geq \pi_n(\varphi)$ for every φ.

Let us now show that $E\left[\min\left\{\psi', \Psi(\varphi)\right\}\right] = \Psi(\varphi) - \int_0^{\Psi(\varphi)} F(\psi)d\psi$:

$$\begin{aligned}
E\left[\min\left\{\psi', \Psi(\varphi)\right\}\right] =& \Pr(\psi' \leq \Psi(\varphi))E[\psi'|\psi' \leq \Psi(\varphi)] + \Pr(\psi' > \Psi(\varphi))\Psi(\varphi) \\
=& F(\Psi(\varphi))E[\psi'|\psi' \leq \Psi(\varphi)] + [1 - F(\Psi(\varphi))]\Psi(\varphi) \\
=& \Psi(\varphi) - F(\Psi(\varphi))\left[\Psi(\varphi) - E[\psi'|\psi' \leq \Psi(\varphi)]\right] \\
=& \Psi(\varphi) - F(\Psi(\varphi))\left[\int_0^{\Psi(\varphi)} (\Psi(\varphi) - \psi)\frac{dF(\psi)}{F(\Psi(\varphi))}\right] \\
=& \Psi(\varphi) - \int_0^{\Psi(\varphi)} (\Psi(\varphi) - \psi)dF(\psi) \\
=& \Psi(\varphi) - \int_0^{\Psi(\varphi)} F(\psi)d\psi \quad \text{(by integration by parts).}
\end{aligned}$$

Substituting the previous expression into equation (A-3), we obtain that the cutoff adjustment factor solves the equation

$$\Psi(\varphi) = \frac{z(\varphi)}{\delta} - \frac{1-\delta}{\delta}\int_0^{\Psi(\varphi)} F(\psi)d\psi. \tag{A-4}$$

To show that the solution is unique, let

$$\mathcal{G}(\Psi(\varphi)) = \Psi(\varphi) + \frac{1-\delta}{\delta}\int_0^{\Psi(\varphi)} F(\psi)d\psi - \frac{z(\varphi)}{\delta}, \tag{A-5}$$

so that $\mathcal{G}(\Psi(\varphi)) = 0$ is equivalent to equation (A-4). We know that $\Psi(\varphi) \in [0, \infty)$ and from equation (A-5) we obtain that $\mathcal{G}(0) = -\frac{z(\varphi)}{\delta} \leq 0$. Note also that $\mathcal{G}(\Psi(\varphi)) \to \infty$ as $\Psi(\varphi) \to \infty$. Therefore, given that $\mathcal{G}(\Psi(\varphi))$ is continuous, there is at least one solution for $\mathcal{G}(\Psi(\varphi)) = 0$ in the interval $[0, \infty)$. Using Leibniz's rule, we get $\mathcal{G}'(\Psi(\varphi)) = 1 + \frac{1-\delta}{\delta}F(\Psi(\varphi)) > 0$ for every $\Psi(\varphi)$. Hence, as $\mathcal{G}(\Psi(\varphi))$ is strictly increasing, the solution is unique. □

Proof of Proposition 2. For part 1, note first that $\pi_n(\varphi) = \pi_o(\varphi) = 0$ for $\varphi \leq \varphi_o$. Then, $z(\varphi) = 0$ if $\varphi \leq \varphi_o$. From equation (A-4), note that if $z(\varphi) = 0$, the equilibrium $\Psi(\varphi)$ solves the equation

$$\Psi(\varphi) = -\frac{1-\delta}{\delta}\int_0^{\Psi(\varphi)} F(\psi)d\psi.$$

As $\Psi(\varphi) \geq 0$ and $-\frac{1-\delta}{\delta}\int_0^{\Psi(\varphi)} F(\psi)d\psi \leq 0$, it follows that the solution is $\Psi(\varphi) = 0$. As ψ is a continuous random variable in the interval $[0, \infty)$, it must be the case that $F(0) = 0$. Therefore, $\Lambda(\varphi) = F(\Psi(\varphi)) = 0$ if $\varphi \leq \varphi_o$.

As $\Lambda(\varphi) = 0$ when $z(\varphi) = 0$, to prove that $\Lambda(\varphi) \to 0$ as $\varphi \to \infty$, it is enough to show that $z(\varphi) \to 0$ as $\varphi \to \infty$. Note that we can rewrite $z(\varphi)$ as

$$z(\varphi) = \frac{\frac{\pi_o \varphi}{\pi_n(\varphi)} - 1}{1 + \frac{f_o}{\pi_n(\varphi)}}.$$

The limit of $\pi_s(\varphi)$, for $s \in \{n, o\}$, as $\varphi \to \infty$ is given by

$$\lim_{\varphi\to\infty} \pi_s(\varphi) = \lim_{\varphi\to\infty}\left[\mathbf{\Omega}\left(\frac{\varphi}{\varphi_s}e\right) - 2 + \frac{1}{\mathbf{\Omega}\left(\frac{\varphi}{\varphi_s}e\right)}\right]\gamma\eta = \infty.$$

Hence, using L'Hôpital's rule we can write the limit of $z(\varphi)$ as

$$\lim_{\varphi\to\infty} z(\varphi) = \lim_{\varphi\to\infty}\frac{\pi_o'(\varphi)}{\pi_n'(\varphi)} - 1.$$

We then get

$$\lim_{\varphi\to\infty}\frac{\pi_o'(\varphi)}{\pi_n'(\varphi)} = \lim_{\varphi\to\infty}\frac{1 - \frac{1}{\mathbf{\Omega}\left(\frac{\varphi}{\varphi_o}e\right)}}{1 - \frac{1}{\mathbf{\Omega}\left(\frac{\varphi}{\varphi_n}e\right)}} = 1,$$

so that $\lim_{\varphi\to\infty} z(\varphi) = 0$.

For part 2, note that $\Lambda'(\varphi) = f(\Psi(\varphi))\Psi'(\varphi)$, where $f(\cdot)$ is the probability density function for ψ. For $\Psi'(\varphi)$, we derive equation (10) with respect to φ and use Leibniz's rule to get

$\Psi'(\varphi) = \frac{z'(\varphi)}{\delta+(1-\delta)F(\Psi(\varphi))}$. Hence,

$$\Lambda'(\varphi) = \frac{f(\Psi(\varphi))z'(\varphi)}{\delta + (1-\delta)\Lambda(\varphi)}.$$

Given that $f(\Psi(\varphi)$ and the denominator are both positive, it is the case that the sign of $\Lambda'(\varphi)$ is identical to the sign of $z'(\varphi)$. I focus then on $z'(\varphi)$.

In the interval (φ_o, φ_n), $\pi_n(\varphi) = 0$ so that $z(\varphi) = \frac{\pi_o(\varphi)}{f_o}$. Thus, with $f_o > 0$ so that $z(\varphi)$ is finite, we have

$$z'(\varphi) = \frac{\pi_o'(\varphi)}{f_o} = \frac{1}{f_o\varphi}\left[\frac{\mu_o(\varphi)}{1+\mu_o(\varphi)}\right]\gamma\eta \tag{A-6}$$

for $\varphi \in (\varphi_o, \varphi_n)$. As $\mu_o(\varphi) > 0$ for $\varphi > \varphi_o$, it follows that $z'(\varphi) > 0$ for $\varphi \in (\varphi_o, \varphi_n)$. Therefore, $\Lambda(\varphi)$ is strictly increasing in the interval (φ_o, φ_n), so that a maximum for $\Lambda(\varphi)$ cannot exist in that region. Hence, if the maximum for $\Lambda(\varphi)$ exists, it must be in the region where $\varphi \geq \varphi_n$. I will prove that this is the case.

If $\varphi \geq \varphi_n$, we get

$$z'(\varphi) = \left[\frac{(\mu_o(\varphi) - \mu_n(\varphi))\,(1+\mu_n(\varphi))\,\gamma\eta}{\varphi\left[\mu_n(\varphi)^2\gamma\eta + (1+\mu_n(\varphi))\,f_o\right]^2(1+\mu_o(\varphi))}\right]\left[f_o - \mu_o(\varphi)\mu_n(\varphi)\gamma\eta\right]. \tag{A-7}$$

Note that if $\varphi = \varphi_n$ (so that $\mu_n(\varphi) = 0$), equation (A-7) collapses to equation (A-6).[21] As $\mu_o(\varphi) > \mu_n(\varphi)$ for every $\varphi \in [\varphi_n, \infty)$, the first term in brackets is always positive. The second term in brackets gives the sign of $z'(\varphi)$ and in particular, it determines the value of φ that maximizes $z(\varphi)$—and hence $\Lambda(\varphi)$. Letting $\hat{\varphi}$ denote the argument that maximizes $z(\varphi)$, it follows that $\hat{\varphi}$ solves the equation

$$f_o - \mu_o(\hat{\varphi})\mu_n(\hat{\varphi})\gamma\eta = 0. \tag{A-8}$$

To show that this is indeed a maximum and that is unique, note that $\mu_o(\varphi)\mu_n(\varphi)$ is strictly increasing in the interval $[\varphi_n, \infty)$ because $\mu_s'(\varphi) > 0$, for $s \in \{n, o\}$. Hence, $z'(\varphi) > 0$ if $\varphi \in [\varphi_n, \hat{\varphi})$, and $z'(\varphi) < 0$ if $\varphi \in (\hat{\varphi}, \infty)$. Note also that given φ_n and φ_o, a lower f_o implies a lower $\hat{\varphi}$ (so that $\mu_o(\hat{\varphi})\mu_n(\hat{\varphi})\gamma\eta$ is smaller). As f_o approaches zero, it follows that $\mu_n(\hat{\varphi})$ must get closer to zero. That is, $\hat{\varphi} \to \varphi_n$ from the right.

For part 3, note that if $f_o = 0$, $z(\varphi) = \frac{\pi_o(\varphi)}{\pi_n(\varphi)} - 1$. Then, $z(\varphi) \to \infty$ if $\varphi \in (\varphi_o, \varphi_n]$ (because $\pi_n(\varphi)$ equals zero). From equation (A-4), it follows that it must be the case that $\Psi(\varphi) \to \infty$ in this interval. Then, $\Lambda(\varphi) = 1$ if $f_o = 0$ and $\varphi \in (\varphi_o, \varphi_n]$. On the other hand, for $\varphi > \varphi_n$, we substitute $f_o = 0$ in equation (A-7) to get

$$z'(\varphi) = -\frac{(\mu_o(\varphi) - \mu_n(\varphi))\,(1+\mu_n(\varphi))\,\mu_o(\varphi)}{\varphi\mu_n(\varphi)^3\,(1+\mu_o(\varphi))}. \tag{A-9}$$

As $\mu_o(\varphi) > \mu_n(\varphi)$, equation (A-9) is always negative. That is, $\Lambda(\varphi)$ is strictly decreasing if $f_o = 0$ and $\varphi > \varphi_n$. $\square$

[21] This implies that as long as $f_o > 0$, $z'(\varphi)$ is continuous.

Proof of Lemma 1. In section 2.1.1, we obtained that $\ln p_i = \ln \hat{p} - \mu_i$. Then, for a firm with productivity φ and technology s, for $s \in \{n, o\}$, we have that $\ln p_s(\varphi) = \ln \hat{p} - \mu_s(\varphi)$. Hence, the average log price of firms with technology s is given by $\overline{\ln p}_s = \ln \hat{p} - \bar{\mu}_s$, where

$$\bar{\mu}_s = \int_{\varphi_s}^{\infty} \mu_s(\varphi) h_s(\varphi \mid \varphi \geq \varphi_s) d\varphi$$

is the average markup of this group of firms. We can then use the expressions for $\overline{\ln p}_n$ and $\overline{\ln p}_o$ in the overall average log price, $\overline{\ln p} = \frac{N_n}{N}\overline{\ln p}_n + \frac{N_o}{N}\overline{\ln p}_o$, to get

$$\ln \hat{p} - \overline{\ln p} = \frac{N_n}{N}\bar{\mu}_n + \frac{N_o}{N}\bar{\mu}_o. \tag{A-10}$$

Now, from equation (3), we can solve for $\ln \hat{p} - \overline{\ln p}$ as $\ln \hat{p} - \overline{\ln p} = \frac{1}{\gamma N}$. Plugging in this result in equation (A-10) we get

$$\frac{1}{\gamma} = N_n \bar{\mu}_n + N_o \bar{\mu}_o.$$

Lastly, substituting in the previous equations our expressions for N_n and N_o from section 2.3, $N_n = (1 - H_n(\varphi_n))(1 - \bar{\Gamma})N_P$ and $N_o = \bar{\Gamma} N_P$, we solve for N_P as

$$N_P = \frac{1}{\gamma \left[(1 - \bar{\Gamma})(1 - H_n(\varphi_n))\bar{\mu}_n + \bar{\Gamma}\bar{\mu}_o \right]}.$$

$\square$

Proof of Lemma 2. We have to prove that $\zeta_\gamma > 0$, $\zeta_\eta > 0$, and that $\zeta_{f_o} < 0$. This is equivalent to proving that $\frac{d\varphi_o}{d\gamma} > 0$, $\frac{d\varphi_o}{d\eta} > 0$, and $\frac{d\varphi_o}{df_o} < 0$, respectively. Taking the total derivative of the free entry condition ($\bar{\pi}_E = f_E$) with respect to γ, we obtain

$$\frac{d\varphi_o}{d\gamma} = -\frac{\frac{\partial \bar{\pi}_E}{\partial \gamma}}{\frac{\partial \bar{\pi}_E}{\partial \varphi_o}},$$

with similar expressions for $\frac{d\varphi_o}{d\eta}$ and $\frac{d\varphi_o}{df_o}$. If equation (17) holds, it is left to show that $\frac{\partial \bar{\pi}_E}{\partial \gamma} > 0$, $\frac{\partial \bar{\pi}_E}{\partial \eta} > 0$, and $\frac{\partial \bar{\pi}_E}{\partial f_o} < 0$.

I show first that $\frac{\partial \bar{\pi}_E}{\partial \gamma} > 0$. Using the expression for $\bar{\pi}_E$ in equation (15), I obtain

$$\begin{aligned}\frac{\partial \bar{\pi}_E}{\partial \gamma} =& \frac{\bar{\pi}_E}{\gamma} + \int_{\varphi_o}^{\infty} \Bigg\{ (\delta + (1-\delta)\Lambda(\varphi))\, \Lambda(\varphi) \left[\frac{E[\psi|\psi \leq \Psi(\varphi)] f_o}{\gamma} - (\pi_n(\varphi) + f_o) \frac{\partial E[\psi|\psi \leq \Psi(\varphi)]}{\partial \gamma} \right] \\ & + [\pi_o(\varphi) - \pi_n(\varphi) - \delta E[\psi|\psi \leq \Psi(\varphi)](\pi_n(\varphi) + f_o)] \frac{\partial \Lambda(\varphi)}{\partial \gamma} \Bigg\} \frac{g(\varphi)}{(\delta + (1-\delta)\Lambda(\varphi))^2} d\varphi. \end{aligned} \tag{A-11}$$

To obtain $\frac{\partial E[\psi|\psi \leq \Psi(\varphi)]}{\partial \gamma}$, note that using integration by parts, we can write $E[\psi|\psi \leq \Psi(\varphi)]$ as

$$E[\psi|\psi \leq \Psi(\varphi)] = \Psi(\varphi) - \frac{1}{\Lambda(\varphi)} \int_0^{\Psi(\varphi)} F(\psi) d\psi. \tag{A-12}$$

Hence, the partial derivative of equation (A-12) with respect to γ is given by

$$\frac{\partial E[\psi|\psi \leq \Psi(\varphi)]}{\partial \gamma} = \frac{1}{\Lambda(\varphi)^2}\left(\int_0^{\Psi(\varphi)} F(\psi)d\psi\right)\frac{\partial \Lambda(\varphi)}{\partial \gamma}.$$

Using equations (A-12) and (10), we rewrite the previous expression as

$$\frac{\partial E[\psi|\psi \leq \Psi(\varphi)]}{\partial \gamma} = \left[\frac{z(\varphi) - \delta E[\psi|\psi \leq \Psi(\varphi)]}{(\delta + (1-\delta)\Lambda(\varphi))\,\Lambda(\varphi)}\right]\frac{\partial \Lambda(\varphi)}{\partial \gamma}. \tag{A-13}$$

Finally, substituting equation (A-13) into equation (A-11), $\frac{\partial \bar{\pi}_E}{\partial \gamma}$ simplifies to

$$\frac{\partial \bar{\pi}_E}{\partial \gamma} = \frac{\bar{\pi}_E}{\gamma} + \int_{\varphi_o}^{\infty}\frac{1}{\gamma}\left[\frac{\Lambda(\varphi)E[\psi|\psi \leq \Psi(\varphi)]f_o}{\delta + (1-\delta)\Lambda(\varphi)}\right]g(\varphi)d\varphi, \tag{A-14}$$

which is unambiguously greater than zero (both components are positive).

For $\frac{\partial \bar{\pi}_E}{\partial \eta}$, we follow the same steps as with $\frac{\partial \bar{\pi}_E}{\partial \gamma}$ and obtain similar expressions: we only to replace γ with η in equations (A-11), (A-13), and (A-14). Hence, we get $\frac{\partial \bar{\pi}_E}{\partial \eta} > 0$.

Lastly, I show that $\frac{\partial \bar{\pi}_E}{\partial f_o} < 0$. Based on equation (15), I get

$$\begin{aligned}\frac{\partial \bar{\pi}_E}{\partial f_o} = \int_{\varphi_o}^{\infty}\Big\{&-(\delta + (1-\delta)\Lambda(\varphi))\,\Lambda(\varphi)\left[E[\psi|\psi \leq \Psi(\varphi)] + (\pi_n(\varphi) + f_o)\frac{\partial E[\psi|\psi \leq \Psi(\varphi)]}{\partial f_o}\right] \\ &+ [\pi_o(\varphi) - \pi_n(\varphi) - \delta E[\psi|\psi \leq \Psi(\varphi)](\pi_n(\varphi) + f_o)]\frac{\partial \Lambda(\varphi)}{\partial f_o}\Big\}\frac{g(\varphi)}{(\delta + (1-\delta)\Lambda(\varphi))^2}d\varphi.\end{aligned} \tag{A-15}$$

As with $\frac{\partial E[\psi|\psi \leq \Psi(\varphi)]}{\partial \gamma}$, I get $\frac{\partial E[\psi|\psi \leq \Psi(\varphi)]}{\partial f_o} = \left[\frac{z(\varphi) - \delta E[\psi|\psi \leq \Psi(\varphi)]}{(\delta + (1-\delta)\Lambda(\varphi))\Lambda(\varphi)}\right]\frac{\partial \Lambda(\varphi)}{\partial f_o}$. Hence, after substituting the previous expression into equation (A-15), we get

$$\frac{\partial \bar{\pi}_E}{\partial f_o} = -\int_{\varphi_o}^{\infty}\left[\frac{\Lambda(\varphi)E[\psi|\psi \leq \Psi(\varphi)]}{\delta + (1-\delta)\Lambda(\varphi)}\right]g(\varphi)d\varphi, \tag{A-16}$$

which is strictly less than zero. □

Proof of Proposition 3. For an increase in γ and η, or for a decline in f_o, we have to prove that there exists a cutoff level, φ^*, such that the offshoring probability, $\Lambda(\varphi)$, declines if $\varphi < \varphi^*$ and increases if $\varphi > \varphi^*$.

With respect to γ and η shocks, it is enough to work with the response of $\Lambda(\varphi)$ to γ, as the derivatives with respect to η are similar (we only need to replace γ with η). Thus, we obtain $\frac{d\Lambda(\varphi)}{d\gamma}$ and derive the conditions that determine its sign.

Similar to the proof of Proposition 2, we obtain that

$$\frac{d\Lambda(\varphi)}{d\gamma} = \left[\frac{f(\Psi(\varphi))}{\delta + (1-\delta)\Lambda(\varphi)}\right]\frac{dz(\varphi)}{d\gamma}.$$

As the term in brackets is positive, we only need to focus on $\frac{dz(\varphi)}{d\gamma}$, where $z(\varphi) = \frac{\pi_o(\varphi) - \pi_n(\varphi)}{\pi_n(\varphi) + f_o}$, and $\pi_s(\varphi) = \frac{\mu_s(\varphi)^2}{1+\mu_s(\varphi)}\gamma\eta$ for $\varphi \geq \varphi_s$ (and zero otherwise). Using the formula for the derivative of the Lambert function in footnote 13, we obtain

$$\begin{aligned}\frac{dz(\varphi)}{d\gamma} = &\left[\frac{(\mu_o(\varphi) - \mu_n(\varphi))\eta}{(\pi_n(\varphi) + f_o)^2(1+\mu_o(\varphi))(1+\mu_n(\varphi))}\right] \\ &\times \left[\mu_o(\varphi)\mu_n(\varphi)\gamma\eta\zeta_\gamma + f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o\zeta_\gamma\right],\end{aligned}$$

where $\mu_s(\varphi) = 0$ if $\varphi \leq \varphi_s$, and is greater than zero otherwise, for $s \in \{n, o\}$. The first term in brackets is non-negative, and strictly positive as long as $\varphi > \varphi_o$. Then, for $\varphi > \varphi_o$, the sign of $\frac{dz(\varphi)}{d\gamma}$ is determined by the sign of

$$\Upsilon_1(\varphi) = \mu_o(\varphi)\mu_n(\varphi)\gamma\eta\zeta_\gamma + f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o\zeta_\gamma.$$

By Lemma 2 ($\zeta_\gamma > 0$), we get that $\Upsilon_1(\varphi) \to -f_o\zeta_\gamma < 0$ as $\varphi \to \varphi_o$ from the right. Also $\Upsilon_1(\varphi) \to \infty$ as $\varphi \to \infty$ (because $\mu_s(\varphi) \to \infty$ as $\varphi \to \infty$ for $s \in \{n, o\}$). Therefore, given that $\Upsilon_1(\varphi)$ is continuous, there is at least one solution for $\Upsilon_1(\varphi) = 0$ in the interval (φ_o, ∞). Given that $\mu_s'(\varphi) > 0$ if $\varphi \geq \varphi_s$, for $s \in \{n, o\}$, it follows that $\Upsilon_1(\varphi)$ is strictly increasing in φ. Therefore, the solution to $\Upsilon_1(\varphi) = 0$, φ^*, is unique. Note that if $\varphi \in (\varphi_o, \varphi^*)$, then $\Upsilon_1(\varphi) < 0$ and $\frac{dz(\varphi)}{d\gamma} < 0$. On the other hand, if $\varphi > \varphi^*$, then $\Upsilon_1(\varphi) > 0$ and $\frac{dz(\varphi)}{d\gamma} > 0$.

For shocks to f_o, we also get that $\operatorname{sgn}\left(\frac{d\Lambda(\varphi)}{df_o}\right) = \operatorname{sgn}\left(\frac{dz(\varphi)}{df_o}\right)$. We get

$$\begin{aligned}\frac{dz(\varphi)}{df_o} = &\left[\frac{(\mu_o(\varphi) - \mu_n(\varphi))\gamma\eta}{(\pi_n(\varphi) + f_o)^2(1+\mu_o(\varphi))(1+\mu_n(\varphi))f_o}\right] \\ &\times \left[\mu_o(\varphi)\mu_n(\varphi)\gamma\eta\zeta_{f_o} - f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o\zeta_{f_o}\right].\end{aligned}$$

Similar to the previous part, the sign of $\frac{dz(\varphi)}{df_o}$ is determined by

$$\Upsilon_2(\varphi) = \mu_o(\varphi)\mu_n(\varphi)\gamma\eta\zeta_{f_o} - f_o(\mu_o(\varphi) + \mu_n(\varphi) + \mu_o(\varphi)\mu_n(\varphi)) - f_o\zeta_{f_o}.$$

By Lemma 2 ($\zeta_{f_o} < 0$), $\Upsilon_2(\varphi) \to -f_o\zeta_{f_o} > 0$ as $\varphi \to \varphi_o$ from the right. Also, $\Upsilon_2(\varphi) \to -\infty$ as $\varphi \to \infty$. Given that $\Upsilon_2(\varphi)$ is continuous, there is at least one solution for $\Upsilon_2(\varphi) = 0$ in the interval (φ_o, ∞). Given that $\mu_s'(\varphi) > 0$ if $\varphi \geq \varphi_s$, for $s \in \{n, o\}$, it follows that $\Upsilon_2(\varphi)$ is strictly decreasing in φ. Therefore, the solution to $\Upsilon_2(\varphi) = 0$, φ^*, is unique. Note that if $\varphi \in (\varphi_o, \varphi^*)$, then $\Upsilon_2(\varphi) > 0$ and $\frac{dz(\varphi)}{df_o} > 0$. On the other hand, if $\varphi > \varphi^*$, then $\Upsilon_2(\varphi) < 0$ and $\frac{dz(\varphi)}{df_o} < 0$. $\square$

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